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Dorota Górecka^{*} Małgorzata Szałucka^{**}

APPLICATION OF MCDA METHODS AND STOCHASTIC DOMINANCE RULES IN THE ENTRY MODE SELECTION PROCESS IN INTERNATIONAL EXPANSION

Abstract

When a company decides to enter overseas markets, it must take a number of strategic decisions, such as, for instance, a decision on the appropriate entry mode. The company has a wide array of choices: various forms of exporting, contractual modes such as licensing, franchising and management contracts, turnkey projects and subcontracting or equity-based modes including wholly-owned subsidiary or joint venture. The various entry modes differ greatly in resource commitment, degree of risk, level of control or profit potential. The appropriate choice of entry mode is a key element of the success of foreign operations and the future of the company. Hence, it is essential for the company to conduct a deliberate and conscious analysis of advantages and disadvantages of each entry mode from the point of view of internal and external factors that influence the choice of entry mode, taking into account the opinion of different participants of the decision-making process.

The aim of this paper is to carry out the simulation of the entry mode selection, using MCDA methods and stochastic dominance (SD) rules, from the perspective of a dynamically developing company that manufactures and distributes hygiene, cosmetic and medical products for women, children and adults.

Keywords: entry mode selection, non-equity modes, equity modes, MCDA methods, EXPROM II with veto thresholds and SD rules.

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1. Introduction

A firm seeking to run its business operation outside its domestic market must make decisions about many related but distinct issues. They are complex and complicated and affect both the likelihood of success and the probability of survival not only of the undertaking abroad, but they may have an additional impact on the success and performance of the internationalizing firm.

The internationalization of the firm has many dimensions. The managers must give careful consideration to many aspects of the process. That is why companies going international should define their entry strategy for international markets in order to perform business functions abroad successfully. International market entry strategy is a comprehensive plan where the company makes decisions about objectives, resources and polices to guide its business operations abroad for a longer period of time to achieve and sustain competitive advantage in the global economy (Root, 1994).

When starting to plan its international market entry strategy, the company must define the reasons why it wants to go abroad. Setting objectives and goals of internationalization has a tremendous impact on the overall strategy determining directions and frames of international expansion. When objectives and goals are set the company must decide on the products or services it wants to deliver to a foreign market. The choice is made in relation to international environment and the company's potential. The next step is to select the target market or markets where the company will sell its products or provide services. It has been recognized widely in the literature as international market selection (Root, 1994; Koch, 2001; Kumar et al., 1994; Cavusgil, 1985; Russow and Okoroafo, 1996; Papadopoulos et al.; 2002; Sakarya et al., 2007; Górecka, Szałucka, 2013).

When the target market is identified, the company must find a way to enter it and launch its products or services. Consequently, it must decide on the entry mode it wants to use to explore the market. Companies have a wide array of entry modes to choose from. The decision about the appropriate arrangements for organizing business activities located outside the home country is a critical part of an entry strategy for international markets (Wind and Perlmutter, 1977; Hill et al., 1990, Kough and Singh, 1988; Agarwal and Ramaswami, 1992). It might have critical implications for the international project's performance (Root, 1994; Woodcock et al., 1994) and its survival (Li, 1995). Finally, when a company knows with what, when and how it intends to expand internationally, it must decide on the timing of the entry.

Since the decision about the internationalization is very complex, the opinion of different persons from different levels of the company's structure (board of directors, managers, experts) is usually taken into account. As regards entry modes, they differ greatly in resource commitment, degree of risk, level of con-

trol or profit potential. Hence, it is essential to conduct an analysis of their advantages and disadvantages from the point of view of a wide variety of internal and external factors and taking into account the opinion of various participants of the decision-making process.

The aim of this paper is to apply multi-criteria decision aiding (MCDA) methods and SD rules to the problem of entry mode selection. Their usefulness will be illustrated by a real-life example of a company that is a leading producer and deliverer of hygiene, cosmetic and medical products seeking new markets.

The paper is organized as follows. Section two focuses on an integrated framework for entry mode selection, presenting possible entry modes to explore international markets and factors that influence the company's choice of entry mode. Section three demonstrates the methodology used in the research including the description of the case study. In section four the research results obtained due to the application of the MCDA methods are presented.

2. A framework for entry mode selection

Among the most critical issues in international market entry strategy is the selection of an appropriate entry mode in order to penetrate the foreign target country. Entry mode has been defined as an institutional agreement that allows the company to enter a market with its products, technology, human skills, management, or other resources (Root, 1994).

A firm entering a foreign market has a variety of mode choices to organize its business activities abroad. Entry modes can be divided into three categories: export entry modes, contractual entry modes and investment entry modes (Root, 1994; Sitek, 2000; Rymarczyk, 2004; Gorynia, 2007; Johnson et al. 2008; Duliniec, 2009). The first category includes indirect and direct export activities. It refers to the manufacture of a product outside the target market and the subsequent shipping of the product to it. Direct exporting can be done via an agent or distributor in the target country or via a direct branch/subsidiary that requires equity investment. Exporting has been considered as the most common way to enter new international markets. Contractual entry modes are understood as nonequity cooperation agreements between a company that wants to enter the market and an entity located in a foreign target market. In contrast to export modes, contractual entry modes involve a transfer of technology or other skills and knowledge between partners. In the case of export modes, the transfer is limited to physical products. The cooperating companies are characterized by their legal autonomy and simultaneous economic interdependence. Firms have a wide array of contractual entry modes to choose from, including licensing, franchising, technical agreements, service contracts, management contracts, turnkey contracts, manufacture contracts and co-production agreements (Root, 1994). The last category – investment entry modes – represents operation modes that are inevitably linked to ownership and equity investment. A firm decides to engage in international expansion by setting up a completely new firm or acquiring an existing local one. An investor may do this alone; maintaining full ownership and control over an affiliate (a branch or a subsidiary) or it may do this with the support of a partner or partners sharing ownership and control. In the literature, the former form of equity-based modes is described as a sole venture and the latter as a joint venture.

Entry modes differ considerably along several dimensions. The most common ones found in the literature are: degree of control (Anderson and Gatignon, 1986; Root, 1994; Kotler, 1994), level of risk (Root 1994; Kotler, 1994) and resource commitment (Hill et al., 1990; Meissner, 1990; Kotler, 1994). Moreover, entry modes have been also characterized by level of management involvement (Meissner, 1990), dissemination risk (Hill et al., 1990), skills requirement (Gronhaug and Kvitastein, 1993) or profit potential (Kotler, 1994). Degree of control, level of risk and resource commitment are highly correlated. Higher control requires higher resource commitment; increased resource commitment leads to higher risk.

The establishment of a wholly owned subsidiary results in the highest level of resource commitment, risk and a level of control, but it also provides the highest level of profit potential and the lowest level of dissemination risk. Joint ventures, where ownership of and responsibility for the management of the operation are shared, is considered as the entry mode with a lower level of resource commitment, control, profit potential and general risk compared to a wholly owned subsidiary, but with a higher level of dissemination risk. In licensing or franchising, the licensee assumes the investment risk – bears the development cost and risk associated with opening up a foreign market, thus the resource commitment and general level of risk is lower than in equity-based modes. At the same time, however, the level of control or economic gains are lower and there is a higher risk that firm-specific advantages in know-how will be expropriated by a licensee. Exporting is characterized by a low level of resource commitment, risk and a level of control.

From the theoretical point of view, entry mode choice is dependent on the analysis of objective information gathered systematically from the environment and the company. In practice, the companies often make their decisions how to enter the foreign market on the basis of non-systematic and ad hoc procedures (Whitelock and Jobber, 2003). This happens due to the highly complex entry mode decision that makes it difficult for the company to make a conscious and deliberate cost/benefit analysis of options.

Entry modes differ significantly in terms of their mix of advantages and drawbacks. The entry mode choice comes down to a trade-off between control and the cost of resource commitment under conditions of certain level of risk (Sarkar and Cavusgil, 1996) which leads to a choice that maximizes riskadjusted return on investment (Anderson and Gatignon, 1986). However the tradeoffs are not easy to evaluate and not well understood. There is still not a comprehensive and easy to apply tool which will allow managers to assimilate a huge amount of information referring to internal and external factors in order to make the right decision about the choice of entry mode. Research in this field is still very fragmented and limited in scope. This paper attempts to provide a comprehensive method to fill in the blanks in this field. Assuming that managers make decisions based on a rational model using the proposed method, they may take into account a wide range of factors influencing entry mode choice and make tradeoffs between each mode in relation to the other relatively easily. However, managers should be conscious of the limitations of the rational decision-making model and of the difficulties with making "optimal decisions". They operate under bounded rationality and make decisions based on incomplete information, under time pressure and under conditions where particularistic goals are contradictory. In reality, their aim is to find the more or less optimal mode at a given point in time. Benito and Welch (1993) emphasize the need for a dynamic approach to foreign entry mode choice. As mentioned above, the entry mode is selected at a given point of time, when specific internal and external conditions prevail. The environment, the company and its strategies evolve over time and the concept of "optimal decision" seems to be unclear from the perspective of the rational models describing the entry mode decision-making process.

A huge range of factors needs to be considered by the company when selecting the most appropriate entry mode for a target foreign market. Managers can be overwhelmed by the diversity and complexity of the required information. In the literature, researchers consider a number of variables to be significant in the decision about the choice of entry mode. Canabal and White (2008) identified around 200 different independent variables used in various entry mode studies. According to their review of empirical studies in international entry mode research, the most commonly used variables were MNE/international experience, cultural distance, risk, firm size, host restriction/host policies (host country variables), R&D intensity, host country experience, industry competition/concentration, size of operation/scale and advertising intensity.

In the context of such a large number of variables affecting the choice of entry mode, researchers suggest to synthesize and group them into sets of variables. There are several proposals for groups of variables that support the assessment process (Root, 1994; Hill et al., 1990; Gannon, 1993; Luo, 1999; Sitek,

2000; Rymarczyk, 2004; Johnson et al. 2008). In this paper we decided to adopt the framework proposed by Root (1994) and we identify four main sets of variables: target country environmental factors, target country industry factors, company factors and company product factors. We strongly believe that home country factors in the case of some countries may be also critical; however, in our case they do not play a significant role. For each group we decided to include the factors commonly referred to in the literature. Their importance in the entry mode decision process is determined mainly by the objectives and goals of company's international expansion and verified by a firm's capacity. When analysing factors, it must be remembered that each of them should be considered in terms of whether it encourages or discourages a particular entry mode.

Target country environmental factors

When making a decision about the right entry mode, managers should pay attention to several host country environmental factors. International entry mode studies confirm their considerable impact on the choice of entry mode. The factors within this group that are considered in the decision process include: market potential, production factors, cultural distance, geographical distance, government policies and regulations of the host country, property rights systems, external economic relations and political risk. All commonly examined factors relate to the macro environment, country attractiveness and market potential.

Market potential (size and growth) has a great impact on the entry mode. It has a direct impact on a firm's size of operation, defining the potential sales volumes. Where market potential is relatively low, we can assume (*ceteris paribus*) that the company will favour entry modes with low resource commitment and low breakeven sales volumes such as indirect exporting, direct exporting via an agent/distributor or contractual arrangements. Otherwise the company may follow an entry strategy with a high resource commitment, such as equity-based modes, finding its justification in high sales potential and in better satisfaction of customers' needs.

One of the reasons for companies going abroad is the presence of resources (production factors) that are not available at home or are of a higher quality and/or lower cost. These factors are considered very widely in the literature and practice. Companies are seeking resources such as natural resources, raw materials, labour, technological, innovatory and created assets (e.g. patents) or physical infrastructure (ports, roads, power, telecommunication). In the majority of cases, when the company wants to exploit these resources, it must be physically present in the host country using investment equity modes. For certain resources, equity-based modes are the only entry modes that can ensure access to them. However,

some of resources may be also exploited indirectly through contractual entry modes. Hence we can assume that the greater benefits from factor endowments in the host country, the more companies will favour solutions that include equity investment.

Cultural distance has been also recognized as a factor affecting market entry mode (Kim and Hwang, 1992; Agarwal, 1994; Brouthers and Brouthers, 2001; Anderson and Gatignon, 1986; Anderson and Coughlan, 1987; Gomes-Casseres, 1990; Erramilli and Rao, 1993). In general, it refers to the distance between the home country and the target country in terms of cultural values, language, social structure or ways of life (Root, 1994). Differences between the countries increase uncertainty and the level of risk as well as the cost of coordinating business operations. We can assume that the greater the cultural distance between the home country and the target country, the more the company will favour nonequity entry modes in order to limit the resource commitment and accompanying risk. Another way for a company to overcome cultural barriers and reduce risk is to involve a local partner or partners who are familiar with the culture of the target country in the economic activity abroad.

Geographical distance has a slightly contradictory impact on entry mode strategies. Greater geographical distances and high transportation costs may significantly deteriorate the company's position compared to its competitors in the target market. The geographical distance also reduces flexibility and the ability to respond quickly to changes in the local market. The greater the geographical distance, the greater the likelihood that firms will decide to make an investment entry. If the geographical distance is low, then export entry may be favoured over other modes (Root, 1994).

The government policies and regulations may also directly or indirectly affect the choice of entry mode. The countries are analysed in terms of how favourable their policies and regulations are to foreign companies willing to enter. High tariffs and tight quotas will hinder exporting activities and encourage companies to locate production in the target country, while a restrictive host country policy on foreign investment will reduce the number of equity investments in favour of other modes such as exporting or non-equity contractual arrangements. In some countries there are legal limits on foreign equity participation in domestic enterprises and companies are forced to operate in the host market using only joint ventures. The host country may offer foreign companies a wide array of incentives in terms of taxation, access to infrastructure, local financing as well as resource or material supply, depending on entry modes favoured by the host country (Luo, 1999).

In this context, external economic relations should also be taken into consideration while selecting the most appropriate entry mode. Exchange rate policy and exchange rate behaviour, the balance of payments, the level of foreign debt

and its service, restrictions on the transfer of capital, profits and salaries etc. should be carefully assessed by managers. Under restrictive exchange controls, companies are better off utilizing low control entry modes such as indirect or agent/distributor exporting or contractual agreements which allow them to reduce negative effects of transfer restrictions. When the exchange rate has depreciated, firms are motivated to produce locally using equity-based entry modes. On the other hand, when the exchange rate has appreciated, export modes are chosen above the other options.

Another aspect of the target country environment concerns property rights systems. This is an essential issue, especially for companies with high technological competences and tacit knowledge. If host countries are unable to ensure effective property rights protection, the company risks leakage or unwanted dissemination of proprietary technological and marketing assets to competitors, suppliers or customers. Faced with potential infringement and piracy by local firms, companies are often willing to select higher ownership modes to reduce the risk of unwanted dissemination. Keeping the transfer and use of intellectual property rights within the company provides the highest level of protection. When property rights protection is sufficient in the host country, companies may select modes offering lower levels of control as the risk of the expropriation of assets is lower. In these circumstances the company does not need to construct a costly governance structures to protect assets.

Finally, political risk is a factor that needs to be examined in order to make the right entry mode decision. In markets where political risk is high, companies try to minimize their resource commitment to ensure strategic flexibility (Anderson and Gatignon, 1986). Flexibility increases the company's ability to exit quickly from the target market without a significant loss when the environment deteriorates. Consequently in markets with high political risk, companies will favour low control and ownership modes. They will tend to use export modes or modes that enable them to share the risk with partners. The most valuable partners will be local, with knowledge about the host country as well as relations that can help to reduce external uncertainty and the impact of a volatile environment. In markets with lower levels of political risk, the companies are more inclined to pursue investment modes such as a wholly owned subsidiary.

Target country industry factors

Various target country industry factors also need to be considered by a firm when entering a new market. The factors within this group considered as part of the decision process include local supply and distribution infrastructure, relations with suppliers and buyers, competitive conditions, demand uncertainty and entry and exit barriers.

When companies enter international markets, knowledge about the availability and quality of local supply and distribution infrastructure in the industry may play a significant role in the process of selecting the appropriate entry mode. Good marketing infrastructure in the target market allows the company to reduce its resource involvement and use an existing network of local agents and distributors to launch products. There is no need to engage deeply in the market with more advanced modes. Indirect and agent/distributor exporting is recommended. Where marketing infrastructure is poor, a branch/subsidiary may be indispensable to reach the local market (Root, 1994). Moreover, when industrial linkages with suppliers and distributors are essential in the industry, it is recommended that the company utilizes high resource commitment modes such as a wholly owned subsidiary or joint venture. Entry modes with partners will be useful when the company does not have industrial linkages and has to build and develop relations with various actors in the industry.

Competitive conditions may lead companies to use high control or low commitment entry modes. One aspect of competitive conditions in an industry is its competitive structure (Root, 1994). When there are many non-dominant competitors in the target market (atomistic structure), the company may prefer to use export entry modes because there is no need for high commitment. In target countries where the competition is oligopolistic or monopolistic, the companies may favour equity investment in production, an option that should improve their ability to compete on the market. However, when competition is too strong for both exporting activities and equity-based modes, Root (1994) recommends licensing or other contractual agreements that allow the company to be present with its products without direct involvement in the market. The other dimension of competitive conditions in an industry is the volatility of competition (Hill et al., 1990). According to Hill et al. (1990), when competition is more volatile companies tend to use low control and ownership modes due to their increased flexibility. Intense competition and rapidly changing environmental factors require from the company the ability to adapt quickly, an ability which is linked with low rather than high resource commitment.

Demand uncertainty is one the most essential factors affecting the entry mode choice. It directly refers to the host country demand for the company's products. If demand is unknown or predicted to be low, there is no point in making a substantial resource commitment (higher resource commitment leads to less strategic flexibility and substantial sunk costs if a withdrawal from the market becomes necessary). Demand conditions vary depending on the stage of the industry life cycle. It has been widely recognized that uncertainty and unpredictability are greatest in the early/embryonic or late/declining stages of the industry life cycle (Vernon, 1966). Thus, when a target market is in its embryonic or de-

clining stage, managers are more inclined to favour low resource commitment and low control entry modes. More stable and predictable demand conditions encourage managers to increase their resource commitment; however, this does not necessarily imply a need for investment modes (Hill et al., 1990).

Entry and exit barriers in the target industry in the host country may also influence a company's choice of entry mode. High barriers reduce a company's freedom to choose from a wide array of available entry modes. It may happen that the company will be forced to accept host government-instituted modes of entry into certain industries (Luo, 1999).

Company factors

When selecting the right entry mode, managers also need to take into consideration some features of the firm they operate. There is a general agreement in the literature that factors such as size of the company, international experience, corporate strategy, generic marketing strategies and nature of the strategic assets are crucial in the entry mode decision-making process.

Firm size has been recognized as an important factor in the entry mode decision process. Sarkar and Cavusgil (1996) highlighted it as one of the key subthemes alongside international experience within firms/foreign venture specific factors. A relationship between firm size and entry mode strategy is a direct reference to resource commitments. As noted above, entry modes differ in terms of resource commitment. Hill et al. (1990) define resource commitment as "(...) dedicated assets that cannot be redeployed to alternative uses without cost (lost value)". We have to remember that with greater resource commitment comes increased risk. Hence small-sized firms will have limited opportunities for international expansion as they must make use of those entry modes requiring resources that are adjusted to their capacity. It must be stressed that resources are understood widely, not only in terms of capital, which may be the first that springs to mind when discussing entry modes, but also in terms of technology, management, marketing and production skills. Small-sized companies often face financial and managerial constraints, forcing them to restrict themselves to the simpler entry modes with low international involvement and resource commitment. Conversely, large firms have lower resource constraints and can bear the higher risk of their international operations. Therefore they can often use more advanced entry modes that offer higher profit potential but also higher risk. An abundance of resources permits the company to limit the consequences of potential failure that could lead a small-sized company to bankruptcy.

International experience is the second key sub-theme within this group of factors. According to Canabal and White (2008), it is the most commonly used variable to explain entry mode choice in empirical studies. Knowledge about

foreign markets and international experience is crucial for increasing involvement in international operations (Johanson and Vahle, 1977). The greater international experience allows the company to reduce risk and uncertainty, which constrain the company's involvement in business functions outside the domestic market. Companies with more experience due to their accumulated market knowledge which have developed capabilities for managing foreign operations are more likely to make higher resource commitments and prefer high-control modes such as a wholly owned subsidiary or joint venture (Gomes-Casseres, 1990). Conversely, the companies with little knowledge and experience in foreign markets face higher levels of exposure to risk. Lack of knowledge or experience may cause errors and inefficiencies. In order to limit exposure to risk, such companies prefer modes offering low-control and low resource commitment, starting with exporting through subcontracting, licensing or franchising. When a company suffers strongly from a lack of local knowledge and experience in the host country, it may tend to prefer modes engaging local partners in business operations in order to gain knowledge and experience in the local market. Hennart (1991), Li (1995), and Delios and Beamish (1999) support a positive relationship between the level of international experience and the level of ownership and control.

Corporate strategy has been also recognized as a factor effecting entry mode choice (Hill et al., 1990; Gannon, 1993; Luo, 1999). The company may pursue one of two basic corporate strategies: a multi-domestic strategy or a global strategy. The assumption on which the multi-domestic strategy is based is that national markets differ widely along many dimensions such as customer tastes and preferences, the competitive and operating conditions, and political, legal, and social structures. In order to meet the different challenges of national markets, companies must confer a high degree of autonomy and responsibility for local activities on national subsidiaries, where the majority of business functions have to be located. A high degree of autonomy for national subsidiaries is a consequence of the need to adapt operations to differing local competitive conditions and products to the specific tastes and preferences of local customers. In general, we can assume that companies pursuing multi-domestic strategy will tend to use modes with a relatively low degree of control and resource commitment to maintain global flexibility and profitability by using entry modes with low breakeven sales volumes. They may also prefer modes involving local partners such as licensing or joint venture in order to limit the resource commitment and gain knowledge and experience in the local market. Conversely, the companies pursuing a global strategy will favour modes with a high degree of control to ensure the effective configuration and coordination of all the activities a company performs all over the world. The basic assumption underpinning a global strategy is a convergence of tastes and preferences among consumers from different national markets. The company sees its sources of advantage over other competitors in the substantial scale economies it can achieve by centralizing production activities and marketing a standardized product to a global market. The national subsidiaries are usually highly specialized units that follow central decisions from headquarters. Under these circumstances, all modes involving partners are not recommended, due to the high level of subordination and low autonomy of national subsidiaries.

Besides corporate strategy, generic marketing strategies are also expected to affect the entry mode decision process (Gannon, 1993; Bradley and Gannon, 2000). One of the strategic decisions the company has to make when entering foreign markets is whether it will pursue a concentration or diversification strategy (Ayal and Zif, 1979). A market concentration strategy assumes a high level of marketing efforts and significant levels of resource commitment to each foreign market in which it operates. It is a consequence of the company's objective to achieve a strong market position in each of its foreign markets. Only when the company achieves a significant share in the foreign market it can enter other new markets. The strategy is based on concentrating resources in a limited number of markets and a slow, gradual increase in the number of markets, country by country. Following a concentration strategy may result in preferring high control entry modes such as wholly owned subsidiaries and joint ventures which are supposed to enable the company to have greater control over strategy and tactics. In contrast, a market diversification strategy assumes a high level of return from low resource commitment in many markets. The company following this strategy is trying to enter many foreign markets within a short period of time. Although this approach permits the immediate penetration of a larger number of foreign markets, it also involves resource dispersion. Hence, following a diversification strategy by the company may result in a preference for low control entry modes and non-equity modes such as indirect exporting, agent/distributor exporting or licensing.

The internationalization theory suggests that the nature of strategic assets also shapes the entry mode decision. High transaction costs associated with a market-based exchange of strategic assets, particularly in the case of firm-specific know-how, result in a positive relationship between the level of control and the specificity of assets (Anderson and Gatignon, 1986, Hill et al., 1990; Delios and Beamish, 1999). In an attempt to avoid the cost of drafting, negotiating, monitoring, and enforcing contracts with economic market actors (with bounded rationality and opportunistic tendencies), companies internalise the transactions within the company's structure. By establishing a wholly owned subsidiary they reduce dissemination risk (risk of losing control) and avoid the market failures related

to information (problems related to the evaluation of those assets by the market). In addition, the internal transfer of assets is considered to be more appropriate and efficient than the market mechanism, when assets, particularly know-how, are tacit and deeply embodied in the company, and it might be problematic to separate it out for a transfer to the partner. Hence we can assume that the more specific and tacit company's assets, the more likely it will choose high-control entry modes.

Company product factors

The last group of factors to which managers should pay attention are factors directly related to the company's product, such as product adaptation, life-cycle stage of the product, levels of customer service, and transaction specificity of the product.

When the company needs to adapt the product to local needs and preferences, it must have considerable knowledge about the local market. Root (1994) indicates that the selected entry mode should assure the company of the close proximity to the foreign market in order to be able to tailor the product to the local customer. An active approach and a deep involvement with the market are essential to fulfil customers' expectations. If so, we can expect that the more customized their products, the more companies are likely to enter a foreign market through high-control entry modes, which seem to be more efficient in this case (Anderson and Gatignon, 1986).

A similar approach to entry mode selection is used in relation to customer service levels. If a product requires pre- and post-purchase service, proximity to the foreign market and customers seems to be crucial. It is hard, sometimes even impossible for the company to fulfil the service requirements at a distance. Thus, we can assume that companies with high service requirements tend to prefer more high-control entry modes in order to achieve the necessary proximity to customers (Lee, 1986).

Life cycle stage of the product (PLC) is related directly to the proprietary content. Anderson and Gatignon (1986) indicate that immature products in the early stages of the PLC are characterized by high proprietary content which generates problems with its transmission and valuation. Moreover, there is a potential risk of loss of technology or property, leading to a need for control. Therefore, the more mature the company's product, the greater the propensity to choose a low-control entry mode.

Transaction specificity of the product (Gannon, 1993; Bradley and Gannon, 2000; Anderson and Gatignon, 1986) is related directly to the nature of the assets that the company possesses. Products of a company might be classified into "high tech" and "high touch" (Levitt, 1983). High tech products are defined as

products with highly intangible components for which objective valuation may be problematic: it is difficult for the buyer to estimate the value of the intangible asset component because it is poorly understood, unless it is disclosed. Those intangible components are related to technological know-how, marketing know-how or brand loyalty (Gannon, 1993) and stand behind the company's technical leadership, product image and reputation or its capacity for fast and flexible response. High touch products are based on tangible assets and are well understood. That's why the objective valuation of them is relatively easy. Thus, when the company possesses highly proprietary products (or processes) it may tend to use entry modes offering greater control due to the hazard of valuation.

3. Methodology

The present study shows the possibility of applying multi-criteria methods from the PROMETHEE family and SD rules to aid decision-makers in the entry mode selection process. It is based on the example of a dynamically developing company that manufactures and distributes hygienic, cosmetic and medical products for women, children and adults. This company is an enterprise with entirely Polish capital, which is organized in 17 countries. The capital group is composed of 54 companies including 17 manufacturing companies (in Poland, Russia, Ukraine and India), 27 trading companies (in 14 European countries, India and the USA) and 10 service (medical and information technology) companies (in Poland and Russia). It employs over 7.3 thousand people and markets its products in more than 65 countries worldwide (they are available on all inhabited continents). Thanks to the firm's own Research and Development Centre that cooperates closely with experienced scientific institutions, its products are based on the cutting edge technologies. This helps the company to compete successfully with international companies in the highly competitive markets in which it operates¹.

The concise history of the firm, emphasizing especially its foreign operations and R&D related activities, is presented in Table 1.

Information about the company comes from its brochure and its website: http://www.tzmo-global.com/en GLO (7 March 2014).

Table 1 Company's history in brief

Years	Event
	The company is established as a state-owned enterprise
	Dressing material is produced for the Ministry of National Defence and the Central Mining Office Supply. Production is set to shut down after completing the order but due to the high
1950s	quality of work further orders appear
	The company begins conquering foreign markets: products are sold in European, African and
	Asian countries
	<u>The company is privatised</u> – a joint-stock company is created by individuals (Polish citizens):
	the employees of the company and representatives of the academic and medical environment
	<u>In 1997 the company receives</u> – as the first firm in Poland – <u>a certificate confirming that it</u>
	produces medical products in accordance with the requirements of GMP (Good Manufacturing
1990s	Practice) – the principles set by the WHO (World Health Organization)
	In addition, the company obtains certificates of conformity of quality management system
	ISO 9001 and ISO 13485
	Since the end of the 1990s the company is entitled to mark its products with the European CE safety mark
	In the early 2000s the company opens a hospital in Poland which – since 2007 – has been serving
	as a modern polyclinic. Since the beginning of 2000s it has been also providing a sterilization
	service for hospitals
	In 2003 R&D company joins the capital group. Thanks to that the offer of the company is
	extended of biomaterials and other technologically advanced products
	Production of hygiene products in the newly built plants in the East market starts - in 2003
	in Russia and in the first quarter of 2004 in Ukraine
	In 2002 the company establishes a joint venture with its Indian partner. A new factory in India
2000s	begins manufacturing hygiene and medical products in 2005. At the end of 2000s it obtains the
	CE mark for medical production
	In 2004 the company <u>builds a modern logistic centre</u> in Poland (which serves as a central distri-
	bution warehouse). The following year a training, marketing and logistics centre is opened
	in Germany. Another logistics centre is founded in 2007 in Romania
	In 2008 new business units are established in Poland (e.g. a films and laminates production plant
	and a clean room for medical production) At the end of 2000s the company starts business activity in North America – it establishes its
	headquarters in the United States
	In 2011 the company finishes work on a modern machine for the production of absorbent pants.
	This is one of the few high-tech machines in the world for the production of absorptive products
	The company consistently develops its business overseas. In 2012 it takes part in the largest trade
2010s	show in the United States for those who are interested in home medical equipment market -
20108	Medtrade
	The company receives many prestigious awards, for instance: Business Eagles in Germany 2011,
	President's Economic Award – 'Polish Economic Nobel Prize' for 'the presence on the global
	market' 2012, 'Orzeł Rzeczpospolitej' for 'the best production company' 2013

Source: http://www.tzmo-global.com/en_GLO/companyHistory (7 March 2014).

The present simulation of an entry mode selection refers to a project already carried out by the company, namely the investment made in India (see Table 1). Hence, it is assumed that the target market had been already selected by the firm.

After considering the various alternatives we have selected six entry modes, which seemed reasonable to apply in the case considered, namely: indirect export, agent/distributor export, licensing, branch/subsidiary export, joint venture and wholly owned subsidiary.

Factors affecting the company's choice of the entry mode have been identified through the literature review. We have selected 15 criteria that should be considered from the point of view of encouraging or discouraging a particular entry mode. They are presented in Table 2.

Table 2 Factors influencing the company's choice of the entry mode

Factors (criteria)	Measures (units)	Evaluation scale
	Target country environmental factors	
	<u>Total population</u> (number of inhabitants)	■ Very low
Market	<u>Urban population</u> (number of inhabitants)	Low Medium
potential	GDP growth rate (annual %) GDP per capita (CDP per conits constant 2000; USD)	■ High ■ Very high
Production factors	(GDP per capita constant 2000; USD) Cotton production (thousand bales) Labour cost (USD per hour)	Low (unattractive)MediumHigh (attractive)
Geographical distance	<u>Distance between capital cities</u> (kilometres)	 Low (up to 1500 km) Medium (from 1500 to 3000 km) High (over 3000 km)
Cultural distance	<u>Cultural distance</u> : power distance, individualism, masculinity, uncertainty avoidance, pragmatism, indulgence (index)	LowMediumHigh
Political risk	Political risk: corruption, government non-payments/non-repatriation, government stability, information access/transparency, institutional risk, regulatory and policy environment (index)	Very lowLowMediumHighVery high

Government policies and regulations	Economic freedom: property rights, freedom from corruption, fiscal freedom, government spending, business freedom, labour freedom, monetary freedom, trade freedom, investment freedom, financial freedom (index)	 Repressed Mostly unfree Moderately free Mostly free Free
	Target country industry factors	
Demand uncertainty	Product-market development: growth rate, number of competitors, competitive structure, technologies, sector access	Birth stageGrowth stageMaturity stageDecline stage
Marketing infrastructure	Outlet density (number per 1,000 inhabitants) Modern Trade density (number of retail stores per million population)	PoorModerateGood
	Company factors	
Size of the company	Employment (number of employees) Sales turnover (thousand PLN)	Small Medium Large
International	Sales on foreign markets (revenue in thousand PLN) Number of markets served	Very lowLowMedium
experience	Number of projects abroad	HighVery high
Corporate strategy	Corporate strategy analysis (based on cost pressure, local responsiveness and global integration)	GlobalMostly globalMostly multi-domesticMulti-domestic
Generic marketing strategies	Generic marketing strategy analysis (based on number of markets and time horizon)	ConcentrationMostly concentrationMostly diversificationDiversification
Nature of the strate-	R&D intensity	■ Low
gic assets	Product technical complexity	MediumHigh
	Company product factors	
Product adaptation	Degree of product customization	Very lowLowMediumHighVery high
Product lifecycle	PLC analysis (based on proprietary content)	Introduction stageGrowth stageMaturity stageDecline stage

Finally, five experts – specialists in the field of foreign investments (two scientists, two practitioners from FMCG sector and one scientist with practical experience) – scored the selected entry modes individually and independently according to their knowledge and experience on scales established by a main expert and taking into account their own evaluation of 15 factors affecting the company's choice of the entry mode.

Table 3 provides the performance matrix for the six entry modes considered and the 15 criteria used to evaluate them.

Table 3 Input data

		1	nput data			
Factors			Entry	modes		
(criteria), scale ²	Indirect Export	Agent/ Distributor Export	Licensing	Branch/ Subsidiary Export	Joint Venture	Wholly Owned Subsidiary
	1	2	2	4	5	5
Maultot notantial	3	5	1	1	2	1
Market potential (1-5)	1	2	2	3	5	4
(1-3)	1	2	1	5	2	5
	1	1	3	2	5	5
	0	0	0	0	1	1
Production	0	0	0	0	0	1
factors	0	0	0	0	1	1
(0/1)	0	0	0	0	0	1
	0	0	1	0	1	1
	1	1	2	1	3	3
Geographical	3	3	1	3	1	1
distance	1	2	3	2	3	3
(1-3)	2	3	2	2	2	1
	1	1	3	2	3	3
	3	4	4	2	3	1
Cultural	1	2	2	3	4	4
distance	2	3	3	3	4	3
(1-4)	1	3	4	1	3	1
	3	4	3	2	3	2
	4	4	4	3	2	1
Political risk	1	1	3	3	4	4
	2	4	4	3	4	3
(1-4)	4	3	4	2	4	2
	3	3	4	2	2	2
C .	4	4	3	2	1	1
Government	3	3	4	4	4	4
policies	2	2	2	1	4	2
and regulations (1-4)	3	3	4	2	4	2
(1-4)	2	3	3	3	2	2
	1	1	1	3	2	3
Demand	2	2	1	3	3	3
uncertainty	2	3	3	2	3	3
(1-3)	2	3	2	1	3	1
	2	3	3	2	2	2

² Higher values indicate that the entry mode is better tailored to the specific situation.

		1	1	1		1
	0	0	0	1	1	1
Marketing	0	0	0	1	1	1
infrastructure	0	1	1	0	1	0
(0/1)	0	0	0	0	1	0
	0	1	1	0	1	0
	1	2	2	3	3	4
Size of the	1	1	1	3	3	4
company	1	1	1	3	4	3
(1-4)	2	2	1	3	3	4
	1	1	2	3	4	4
	1	2	2	4	3	4
International	1	1	1	2	2	4
experience	1	1	1	3	4	2
(1-4)	1	1	2	3	3	4
	1	1	2	2	3	4
	1	2	1	3	2	3
Corporate	1	1	1	2	3	3
strategy	1	1	1	3	2	3
(1-3)	1	1	2	1	2	3
, ,	1	1	2	3	2	3
	1	1	1	3	2	3
Generic	1	1	1	2	2	3
marketing	1	2	1	2	2	3
strategies	1	1	1	2	1	3
(1-3)	1	2	2	3	2	3
	1	1	1	3	2	2
Nature of the	1	1	1	2	2	3
strategic assets	2	2	2	2	2	2
(1-3)	1	1	2	2	1	3
	1	1	2	2	2	3
	1	1	1	4	2	3
Product	1	1	1	2	3	3
adaptation	1	1	1	2	3	3
(1-3)	1	2	2	1	3	1
	1	1	1	3	1	2
	1	1	1	3	2	3
	1	1	1	2	2	3
Product lifecycle	1	2	2	3	3	3
(1-3)	1	2	1	2	1	3
	1	1	2	2	1	3
	1	1			1	3

To rank entry modes from the best to the worst from the point of view of the expansion of the considered company to the Indian market, the PROMETHEE II method (see Brans and Vincke, 1985; Brans, Vincke and Mareschal, 1986) with SD rules and veto thresholds (see Nowak, 2005; Górecka 2009) and the EX-PROM II method (see Diakoulaki and Koumoutsos, 1991) with SD rules and veto thresholds (see Górecka, 2010; Górecka 2011) have been applied.

Although expected utility models and outranking relation models used to be often treated as competitors, it is possible to benefit from both approaches in the situation when the performances of various alternatives are evaluated in a probabilistic way (as it is in this case because the number of experts participating in

evaluation is greater than one). Namely, stochastic dominance rules can be employed to establish preferences with respect to each criterion and the criteria aggregation method based on the outranking relation procedure can be used to obtain global preference (Martel, Zaraś, 1995). Moreover, the concept of pseudo-criteria can be employed to distinguish situations of strict preference, weak preference and indifference (Nowak, 2004). As a matter of fact, applying this combined approach seems to be an appropriate solution in the case of entry mode selection.

The following characteristics of the decision-making problem analysed and the following expectations of the decision-makers should be taken into consideration in the process of selecting the most appropriate multi-criteria decision aiding method for the problem of choosing the most proper entry mode:

- the decision-making problem should be formulated as a problem of ordering a finite number of alternatives;
- the problem is a group decision-making problem experts engaged in the entry modes' appraisal evaluate them individually and independently and it is required to incorporate diverse individual views into a blended final decision;
- decision-makers are able to present the information about their preferences but they do not have much time for interaction and cooperation with the analyst;
- participants of the decision-making process have very diverse educational background and their knowledge about multi-criteria decision aiding methods is usually limited;
- the decision aiding technique should not be too complicated to enable decision-makers to understand how it works;
- it should be taken into account that experts appraising entry modes may not be consistent in their evaluations, especially in view of uncertainty and inaccuracy characteristic for the decision-making problem discussed;
- the possibility of the occurrence of complete compensation should be removed as in the case of some criteria it may be hazardous;
- it is desired that the final solution takes the form in which the points occur, otherwise it may be unconvincing for the decision-makers.

Taking into account all the above-mentioned information the most suitable methods to aid the decision-making process seem to be PROMETHEE II and EXPROM II with SD rules and veto thresholds. They are considered to be user-friendly, i.e. simple and easily understood – all steps can be quite effortlessly explained to the decision-makers as they are neither very complex nor mathematically challenging. Additionally, thanks to the introduction of the veto threshold the techniques are partially compensatory (a really bad score on one criterion cannot be compensated with a good score on another). Moreover, these techniques allow us to obtain a complete pre-order of the alternatives to which the points are assigned in the final solution. When comparing both methods, the

PROMETHEE IIv method with SD rules results in an ordinal scale of measurement, while the EXPROM IIv method with SD rules, which is based on the notion of ideal and anti-ideal solutions, enables the decision-maker to rank alternatives on a cardinal scale.

To check the influence of changes in the weights of evaluation criteria on the final rankings of entry modes examined the analyst in cooperation with the main expert have established four different vectors of weighting coefficients. The first vector was determined arbitrarily, the second one was created with the help of the AHP method (Saaty, 2006; Saaty and Vargas, 1991), and the third one used Hinkle's method, which is also called the 'resistance to change' grid (Hinkle, 1965; Rogers and Bruen, 1998). In the last approach all factors were presupposed to be equally important. The analyst and the main expert established also the values of indifference (q), preference (p) and veto (v) thresholds. The model of preferences for the decision-making problem is presented in Table 4.

Table 4 Model of preferences

Factors	Max	Vect	Vectors of weighting coefficients					
(criteria)	/min	I	II	III	IV	q	р	V
Market potential	max	0.11	0.1379	0.140	0.067	0	1	3
Production factors	max	0.11	0.1379	0.140	0.067	0	0	1
Geographical distance	max	0.04	0.0305	0.013	0.067	0	1	5
Cultural distance	max	0.06	0.0520	0.070	0.067	0	1	5
Political risk	max	0.09	0.0861	0.100	0.067	0	1	3
Government policies and regulations	max	0.04	0.0305	0.013	0.067	0	1	5
Demand uncertainty	max	0.09	0.0861	0.100	0.067	0	1	2
Marketing infrastructure	max	0.06	0.0520	0.070	0.067	0	0	1
Size of the company	max	0.09	0.0861	0.100	0.067	0	1	3
International experience	max	0.11	0.1379	0.140	0.067	0	1	3
Corporate strategy	max	0.06	0.0520	0.070	0.067	0	1	5
Generic marketing strategies	max	0.02	0.0195	0.005	0.067	0	1	6
Nature of the strategic assets	max	0.04	0.0305	0.013	0.067	0	1	5
Product adaptation	max	0.04	0.0305	0.013	0.067	0	1	5
Product lifecycle	max	0.04	0.0305	0.013	0.067	0	1	5

4. Results

Tables 5 and 6 provide, respectively, a summary of the results obtained by applying the PROMETHEE IIv and EXPROM IIv techniques with SD rules using four different vectors of weighting coefficients.

Table 5
Rankings of the entry modes obtained using PROMETHEE II with veto thresholds and SD rules for four different vectors of weights

NI.	PROMETHEE II with veto thresholds				
No.	Vector no. 1	Vector no. 2	Vector no. 3	Vector no. 4	No.
1	Joint Venture	Joint Venture	Joint Venture	Joint Venture	1
,	Wholly Owned	Wholly Owned	Wholly Owned	Wholly Owned	2
2	Subsidiary	Subsidiary	Subsidiary	Subsidiary	Z
3	Branch/	Branch/	Branch/	Branch/	3
3	Subsidiary Export	Subsidiary Export	Subsidiary Export	Subsidiary Export	3
4	Licensing	Licensing	Licensing	Licensing	4
_	Agent/ Distributor	Agent/ Distributor	Agent/ Distributor	Agent/ Distributor	_
5	Export	Export	Export	Export	5
6	Indirect Export	Indirect Export	Indirect Export	Indirect Export	6

Table 6
Rankings of the entry modes obtained using EXPROM II with veto thresholds and SD rules for 4 different vectors of weights

No.	EXPROM II with veto thresholds						
140.	Vector no. 1	Vector no. 2	Vector no. 3	Vector no. 4	No.		
1	Wholly Owned	Wholly Owned	T * 4 X7 4	Wholly Owned	1		
1	Subsidiary	Subsidiary	Joint Venture	Subsidiary			
2	Joint Venture	Joint Venture	Wholly Owned Subsidiary	Joint Venture	2		
2	Branch/	Branch/	Branch/	Branch/	•		
3	Subsidiary Export	Subsidiary Export	Subsidiary Export	Subsidiary Export	3		
4	Licensing	Licensing	Licensing	Licensing	4		
_	Agent/Distributor	Agent/Distributor	Agent/Distributor	Agent/Distributor	_		
5	Export	Export	Export	Export	5		
6	Indirect Export	Indirect Export	Indirect Export	Indirect Export	6		

The rankings presented in Tables 5 and 6 show the robustness of the solutions to the changes in the vectors of weights as the modifications of the parameters' values do not lead (with only one exception) to alterations in the rankings of entry modes.

The rankings of the entry modes we have obtained are not in complete agreement. The best entry mode, taking into account its appropriateness as the institutional agreement allowing the considered company to enter the Indian market, is joint venture or wholly owned subsidiary. Branch/subsidiary export also turned out to be quite a good solution – the values of net flows determined for it are in all cases positive. In turn, licensing and agent/distributor export do not seem appropriate arrangements for organizing business activities in India by the company examined as the values of net flows determined for them are in all cases negative. Finally, the worst mode to enter the Indian market is indirect export.

To sum up, taking into account all the results obtained, joint venture is recommended for the analysed company (top-ranked five times). Above and beyond, the firm may consider wholly owned subsidiary (top-ranked three times) or branch/subsidiary export as the entry modes to explore the Indian market.

5. Conclusions

In the paper we have proposed a universal tool, based on the outranking MCDA methods combined with stochastic dominances, namely PROMETHEE II with SD rules and veto thresholds, and EXPROM II with SD rules and veto thresholds, which can be used to solve the entry mode selection problem for international expansion. In fact, applying this approach can enhance the evaluation process and improve decision-making since the assumptions on which it is based are in line with reality. The usefulness of the presented tool is confirmed by the fact that in reality, the firm that formed the basis of our analysis of its international expansion chose joint venture as the entry mode to explore the Indian market and it has succeeded on it.

The approach discussed can be applied to any company searching for a way to enter the target market and launch its products or services. Nonetheless, the criteria and measures should certainly be tailored to each firm's specific circumstances and challenges. The example presented in the paper may serve as guidelines to other companies.

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REMARKS ON DESIGNING ITERATIVE MULTICRITERIA PROCUREMENT AUCTIONS

Abstract

In this paper some mechanisms of the multicriteria procurement auctions are discussed, including the elements of the decision support to the auction organizer, as well as to the bidders. The auction mechanisms are considered in the context of attaining incentive compatible decisions. Using domination relations formulated in the criteria space, different rules for the improvement of offers in successive rounds of the auction process are analyzed. The general discussion is illustrated by an example of an iterative multicriteria closed-bidding auction conducted with the use of a multi-agent computer-based system. The system supports submission of offers, multicriteria analysis performed by the auction organizer, simulation, and analysis of the competing bidders' behavior. Experimental results of sessions conducted with the use of the system are analyzed.

Keywords: Multicriteria auctions, incentive compatible decision mechanisms, multiagent systems, multicriteria optimization.

1. Introduction

For auctions with scalar valuation of offers (valuated by price only), there exists a rich bibliography dealing with auction theory, including the papers by Klemperer (2004); Milgrom, Weber (1982); Vickrey (1961). In such auctions we have to select a single offer with the best price. In the case of the iterative multicriteria

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auctions, in each round we have to deal with a set of promising offers valuated by the auction organizer with the help of a vector of criteria. It is reasonable to support multicriteria analysis performed by the auction organizer and to construct an auction mechanism that could lead to the best final offer, according to his true preferences. In most papers dealing with multicriteria auctions, aggregation models are applied, by aggregating multiple criteria to a scalar value using a vector of weights (see De Smet, 2007; Teich et al., 2006; Bichler Kalagnanam, 2005). In this case, the auction organizer has to reveal his model of preferences. Interesting are papers using the reference point approach of multicriteria optimization (Ogryczak, Kozłowski, 2011; Bellosta et al., 2004). This paper belongs to the last class.

The research presented here is a part of a wider research trend dealing with analysis of incentive compatible multicriteria decision mechanisms. Within this research, decision situations are analyzed where there is a number of independent agents that have private information and act according to their own interests. Each agent tries to achieve his own multiple egoistic goals, but the results depend on the actions of other agents. Our research subject includes investigation of the multicriteria decision mechanisms that could lead to incentive compatibility, by revealing true multiobjective preferences and by appropriate coordination of agents' activities, so that the efficiency of the whole process can assured. The incentive compatibility of the multi-commodity market mechanisms was analyzed previously by Toczyłowski (2003; 2009). The ideas developed in these papers have inspired the present study. An analysis of the incentive compatible multicriteria decisions has been presented in Kruś, Skorupiński, Toczyłowski (2012) for a particular case of the producer and buyers problem.

This paper deals with a multicriteria closed bidding-auction for procuring an object, realized in one or in many rounds. This is a case of the multicriteria reverse-type auction. Different forms and rules of the auction are analyzed that are not limited to the current rules of the public auctions defined by law. The auction organizer (buyer) and bidders make multicriteria decisions. The organizer and bidders have private knowledge about their own preferences and possibilities. This information is confidential. The organizer minimizes criteria (such as cost, realization time, etc.). Bidders know these criteria, but the organizer does not inform them about his preferences.

In the classic reverse type auction, we have a sequence of offers proposed by the bidders, with gradually decreasing prices. Each bidder has his reservation price (see Figure 1a). It is obvious that any possible contract below his reservation price is not profitable for him. The organizer has also his reservation price that defines an upper limit for the price he can accept. Information about the reservation prices is private and confidential.

In the multicriteria auction, possibilities of each bidder are defined by his profitability limits that can be presented in the criteria space formulated by the organizer. These limits restrict possible offers of the bidder (from below). The organizer has also his profitability limit of acceptable offers. An illustration of the profitability limits for two criteria – time and cost – are presented in Figure 1b. Information about profitability limits is private and confidential.

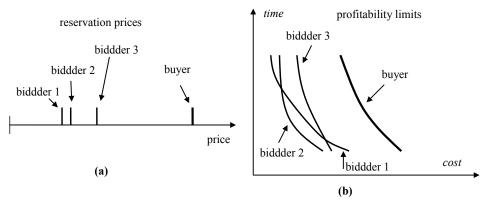


Figure 1. Examples of private information in auctions, (a) reservation prices in the classic reverse auction, (b) profitability limits in multicriteria auctions

In the multicriteria auction design, there are still some open questions regarding the rules for improvement of offers in consecutive rounds, range of information accessible to bidders, form of multicriteria decision support, and others. Regarding incentive compatibility of the multicriteria decisions, an interesting question arises: to what extent the auction mechanism can reveal private information of the bidders to the organizer, to attain efficiency of the allocation.

In this paper a general scheme of the multicriteria auction mechanism is discussed, including elements of the decision support to the auction organizer as well as to the bidders. Using domination relations formulated in the space of multiple criteria, different rules describing improvement of offers in the successive rounds of the auction process are analyzed. The general discussion is illustrated by an example of an iterative multicriteria closed-bidding auction conducted with the use of a multi-agent computer-based system. The system (Skorupiński, 2010; Kruś, Skorupiński, Toczyłowski, 2013) supports submission of offers, multicriteria analysis performed by the organizer of the bidding auction, simulation, and analysis of competing bidders' behavior. Experimental results of sessions conducted with the use of the system are presented and analyzed.

2. Problem formulation

Let a decision making authority organizes a procurement auction for the construction of a facility. We assume that there is a set of n bidders competing to obtain the order for the construction. Denote by $O = \{o^1, o^2, ..., o^n\}$ the set of bidders participating in the auction. The offers $x \in X$, where X is a set of admissible offers, are valuated by a vector of m criteria $y = \{y_1, y_2, ..., y_m\} \in \mathbb{R}^m$ defined by the auction organizer, further called also the buyer. Let $W: X \to \mathbb{R}^m$ be a mapping assigning a vector of the criteria to each offer. The buyer would like to obtain the best offer with the minimal values of the criteria.

We define two relations in \mathbb{R}^m : weak domination: $y^1 \succeq y^2 \Leftrightarrow y_i^1 \leq y_i^2$, for each i = 1, 2, ..., m, and domination: $y^1 \succ y^2 \Leftrightarrow y_i^1 \leq y_i^2$, $y^l \neq y^2$, for each i = 1, 2, ..., m, where $y^1, y^2 \in \mathbb{R}^m$.

The buyer knows the profitability limits, defined as the set of acceptable offers X^0 and, related to them, the set of acceptable multicriteria valuations $Y^0 = W(X^0)$. The offers not belonging to the set X^0 are not accepted by the buyer.

The auction is conducted iteratively, in rounds t = 1, 2,... In round t, the bidders present their offers $x^{i}(t)$, where i = 1, 2,...n is the index of a bidder. Each bidder i has also his own profitability limits, defined by the set of admissible offers X^{i} and the related set of multicriteria valuations $Y^{i} = W(X^{i})$. If the bidder cannot find a new offer in the set, which would beat the offers submitted so far, he gives up and cannot continue the bidding.

A general scheme of the auction carried out with use of a computer-based system is presented in Figure 2. The actions of the system operator and the decision-making processes of the auction organizer (buyer) and bidders are taken into account. The system operator starts the session and activates the computer agents that support the organizer and the bidders. The organizer specifies the requirements of the order for the construction of a facility. The specification is presented to the bidders. Before the bidding starts, the organizer and the bidders should define their profitability limits and the corresponding sets: the set of offers acceptable by the organizer and the sets of offers admissible for the bidders. The information about the profitability limits and about the sets is private and strictly confidential.

The organizer starts the first round of the auction. The bidders prepare and present their offers. The organizer collects the offers and analyzes them by looking for the preferred ones. He can either finish the bidding process or start the next round of auction. At the start of the new round the bidders obtain information about the previous non-dominated offers. Then they can prepare and submit new improved offers, which are again analyzed by the organizer.

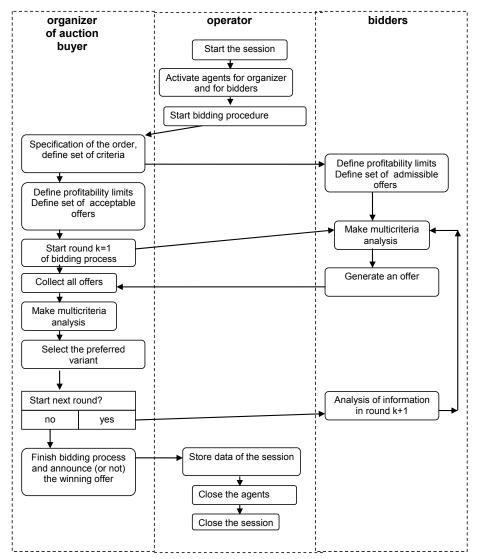


Figure 2. General scheme of the decision making processes in a multicriteria auction

The auction organizer – buyer – would like to obtain an offer that is the best with respect to his preferences. On the other hand, each bidder would also like to obtain a contract which satisfies his profitability limits and is the best with respect to his preferences.

In the case of the classic reverse auction, in successive rounds bidders propose offers with gradually decreasing prices. In the case of the multicriteria auction, in each round there can be a set of offers proposed by bidders which can be non-comparable in the sense of the domination relations mentioned above. Since the buyer performs multicriteria analysis in each round, it is necessary to support the analysis. An example of a set of offers analyzed by the buyer is presented in Figure 3, as a set of black dots in the space of two criteria y_1 , y_2 . In the set there are non-dominated (Pareto-optimal) points, from the point of view of the buyer, denoted by y^1 , y^2 , y^3 , y^4 , y^5 , y^6 (see Figure 3c).

Multicriteria analysis of the set of offers and selection of the offer according to the preferences of the buyer can be done with the use of the reference point approach (Wierzbicki, 1986; Wierzbicki, Makowski, Wessels, 2000). The method has been used and implemented in the computer-based system constructed for experimental studies on a multicriteria bidding auction (Kruś, Skorupiński, Toczyłowski, 2013). The reference point method has been originally developed for the analysis of offers in multicriteria auction by Ogryczak & Kozłowski (2011).

3. Remarks on multicriteria auction mechanisms

Some questions arise regarding the rules of auction and the range of information accessible to bidders in each round. The rules of seeking improved offers can be formulated in different ways. We consider the following three variants:

- a) the new offer is accepted if it cannot be dominated by offers given in previous rounds,
- b) the new offer dominates at least one offer non-dominated in the previous round,
- c) the offer proposed should dominate a non-dominated offer selected by the buyer in the previous round.

Figure 3 presents the sets of possible improved offers in variants **a**, **b**, **c**, as shadowed areas.

Variant a defines the weakest requirements regarding offers that can be submitted in each successive round. Each bidder can propose an offer which dominates an offer non-dominated in the previous round, but can also propose an offer noncomparable with the offers non-dominated in the previous round. The set to which the improved offers should belong is constructed as the sum of the shifted domination cones without their borders.

In variant **b** each proposed offer should dominate at least one offer non-dominated in the previous round. The set which defines the possible improved offers is constructed as the sum of the domination cones shifted to the points representing offers non-dominated in the previous round. Some offers which could be proposed in variant **a** cannot be proposed in this variant, though they could be of interest to the buyer. In variants **a** and **b**, the bidders should have information about all non-dominated offers proposed in the previous round. The buyer does not need to state which of the non-dominated offer he prefers, however, such information could speed up the auction process.

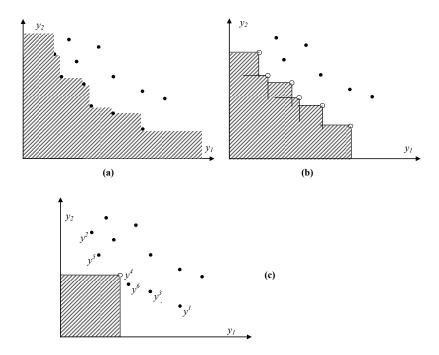


Figure 3. Sets of possible offers according to rules (a), (b), (c)

In variant **c** the buyer, after each round, informs the bidders about his preferred offer and expects that at least one of his criteria will be improved. This variant defines the strongest requirements regarding the offers proposed in the successive rounds. The auction process is speed up in comparison with variants **a** and **b**. On the other hand, some offers, which are non-dominated and of interest to the buyer, could be omitted. This is important especially at the end of the auction, when the bidders are close to their profitability limits.

Figure 4 presents sets Y^1 , Y^2 , Y^3 of admissible offers of three bidders in the space of criteria y_1 , y_2 of the buyer. These sets correspond to the profitability limits of the bidders. Black dots represent offers given in round t-1. The offer y(t-1) represented by a small circle has been selected by the buyer as the preferred one in round t-1. At this place starts the set of offers that can be proposed by the bidders in round t according to variant \mathbf{c} . This set is the domination cone shifted to the point y(t-1). Black diamonds represent offers given in round t. The offer y(t) represented by a small circle has been selected by the buyer as his preferred one in round t. At this point starts the set of possible offers in the next round. The sets of offers that can be proposed by the bidders are limited by their profitability limits and are decreased in successive rounds. Finally,

individual bidders have to interrupt the auction and some offers that could be of interest to the buyer can be omitted. This results from the rule which defines the improvement of offers assumed in variant \mathbf{c} .

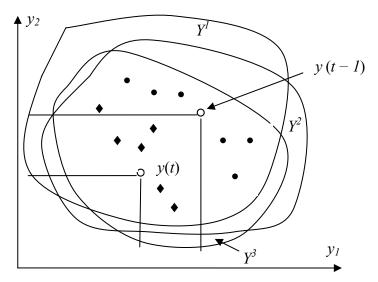


Figure 4. Sets of admissible offers. Examples

When the auction mechanism is constructed, different rules can be used at different stages of the auction process. For example, variant **c** can be assumed as the basic one. However, at the beginning and in the final rounds, variant **a** or **b** can be applied instead. At the beginning of the auction the buyer is not fully aware of his preferences, therefore the bidders should have an opportunity to present a wide portfolio of offers, and this is enabled by variants **a** and **b**. Similarly, in the final rounds it would be a pity to miss some offers, which are non-dominated and lie near the border of the domination cone, excluded from consideration by variant **c**.

The questions discussed above have been solved in a specific way in the case of a closed bidding-auction analyzed in our study. Let us assume that a decision making authority organizes an auction for the construction of a public facility, for example a bridge. The authority is interested in constructing the facility in the shortest time and with the lowest cost possible. The authority – the auction organizer and buyer, defines a discrete set T of several construction time variants, with realization times $tr \in T$. We assume that the organizer and each bidder have their own profitability limit for each time variant. For the organizer, it is the maximal accepted cost of realization of the object. For the bidders, it is assumed that each of them has conducted multicriteria analysis of the possible realization

of the facility. On this basis, the organizer has defined values of minimal payments for the facility construction for the time variants. For lower values, the construction of the facility is not profitable for him. Confidentiality of information is approved. The bidders do not know which time variant will be finally accepted by the organizer. No bidder knows the profitability limit of the organizer or the profitability limits of the competitors. The organizer does not know the profitability limits of the bidders. The auction mechanism should lead to finding the contractor and the best variant of project realization according to the preferences of the organizer.

A special multiagent system has been constructed to simulate different variants of a bidding auction. The system has been written in the AIMMS (see Bisschop, Roelofs, 2009) environment. Users play the roles of the organizer of the auction and of the bidding competitors. The system is started by an operator who initiates actions of a computer agent acting for the organizer and setting a required number of agents for the competitors. The system supports confidentiality of information. The auction is carried out according to the general scheme presented in Figure 2. In each round, the bidders can present their offers with prices for each time variant. The organizer performs multicriteria analysis of the offers submitted. He does not inform the bidders about his preferences. They obtain information about the best offers for each time variant, but do not know who has proposed these offers.

Multicriteria analysis is performed by the organizer interactively with use of the reference point method developed by A.P. Wierzbicki (Wierzbicki, 1986; Wierzbicki, Makowski, Wessels, 2000). According to this method, the organizer can find and analyze non-dominated offers in the space of his criteria, assuming the reservation points r and the aspiration points a in this space. The subscripts a of the components a of vectors a and a refer to the cost and the time, respectively, of the project realization. A set of the indexes is denoted by a, in our case a = {cost, time}. The following optimization tasks are solved:

$$\max z + \varepsilon \sum_{i \in I} z_i$$

subject to the constraints of the reference point method:

$$\begin{split} &z \leq z_i, \forall i \in I, \\ &z_i \leq \gamma(x_i - r_i)/(a_i - r_i), \forall i \in I, \\ &z_i \leq (x_i - r_i)/(a_i - r_i), \forall i \in I, \\ &z_i \leq \beta(x_i - a_i)/(a_i - r_i) + 1, \forall i \in I, \end{split}$$

to the limits for minimized values for the time and the cost:

$$\begin{aligned} x_{cost} &\geq p_{o,tr} - (p_{\text{max}} - p_{\text{min}})(1 - w_{o,tr}), \forall o \in O, tr \in T, \\ x_{time} &\geq d_{tr} - (d_{\text{max}} - d_{\text{min}})(1 - q_{tr}), \forall tr \in T, \end{aligned}$$

and to the constraints related to the discrete form of the set T:

$$\begin{split} & \sum_{o \in O, t \in T} w_{o, tr} = 1, \\ & \sum_{o \in O} w_{o, tr} = q_{tr} \forall tr \in T. \end{split}$$

In this formulation there are additional variables z, z_{cost} $z_{time} \in \mathbb{R}^1$, and the coefficients of the reference point method ε , β , γ , where ε is a small positive number; $0 < \beta < 1 < \gamma$; p_{max} and p_{min} are the most costly and the cheapest offer for the given time variants; d_{max} and d_{min} are the shortest and the longest realization time; $w_{o,tr}$ for $o \in O$ and $tr \in T$, q_{tr} for $tr \in T$ are additional binary variables.

This is a mixed integer-programming problem. The reference point method is implemented for the considered multicriteria optimization problem of the auction organizer. The problem is solved by the system for the points r and a, set by the organizer. The solution of the problem – the point x in the criteria space – is non-dominated in the set of variants proposed by the bidders, due to the properties of the reference point method. The organizer can obtain a representation of the set of the non-dominated offers by changing the reference points.

The organizer finishes multicriteria analysis after having valuated and compared all non-dominated points of interest for him. Then he either selects the best solution, according to his preferences and announces the selected offer, completing the bidding auction, or decides to continue the auction for the next round.

If he decides to continue the auction, the bidders obtain information about the cheapest offers for the indicated time variants. However, they do not know which bidder has presented a given offer, and they do not know the preferences of the organizer. Each bidder can update his offers by decreasing costs. He cannot, however, retract his previous offer unless he wants to correct it. Moreover, he does not know if the auction will be continued in the next round. The organizer opens new offers and repeats the multicriteria analysis with the new set of offers. He can continue the process in the next round; he can either stop the process at any round and interrupt the auction if he has found all the offers unsatisfactory, or can complete the auction announcing the selected offer.

A number of simulated interactive auction sessions have been made with the use of the computer-based system. Human users of the system played roles of an auction organizer and of bidders. We wanted to investigate possible behavior of the organizer and of the bidders. An important question can be posed, whether a multi-round and multicriteria auction mechanism encourages to reveal some confidential information of the bidders about their true cost of realization of the public facility.

4. Experimental results

Selected results of one of the sessions are presented and analyzed below. The session is related to a bidding auction for the construction of a public facility. Three bidders have participated in it. The auction organizer has defined six possible time variants for the realization of the contract: 30, 33, 36, 39, 42 or 45 months. He has also defined his profitability limit, i.e. the maximal cost he can pay for the project realization for each time variant. We assume that each bidder has also defined his profitability limit i.e. the lowest price for which he can construct the facility in each given time variant.

The profitability limits of the organizer and of the bidders are presented in Figure 5. In this example, the profitability limits of bidders are below the profitability limit of the organizer. There exist cost intervals in which the possible solutions of the auction can be profitable for both the organizer and the winning bidder. A comparison of these profitability limits is presented here for the purpose of our analysis only. The organizer does not know the profitability limits of the bidders, and the bidders do not know the profitability limit of the organizer.

The organizer is interested in the construction of the facility in the shortest time possible and at a minimal cost. He understands that the construction of the facility in a shorter time requires a greater cost.

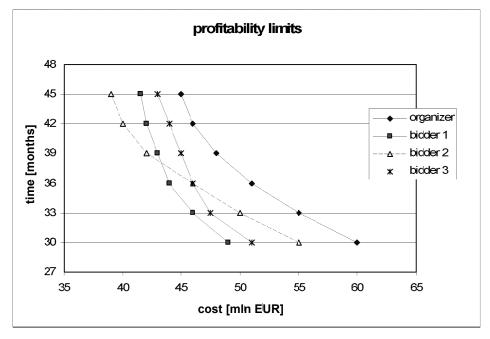


Figure 5. Profitability limits of the organizer and of the bidders

In each round, the organizer performs multicriteria analysis after all offers have been collected. The analysis consists of a number of iterations of the reference point method. In each iteration, the organizer assumes a reservation point and an aspiration point in his criteria space. The computer-based system solves the optimization task formulated in the previous section and derives the corresponding non-dominated point. The organizer obtains a representation of the set of non-dominated points by assuming different aspiration and reservation points, and can then select the best point, which is close to his preferences, but he informs the bidders about the decision after having decided to end the auction.

Figure 6 presents offers in the final (fourth) round. If the realization time is equal to 30, 33, or 36 months, the best offers are those of bidder 1, while in the case of 39, 42 or 45 months, the best offers are those of bidder 2. The organizer has obtained a significant improvement of offers in comparison with the best initial offers given in round one. Concurrent offers have been shown for each time variant.

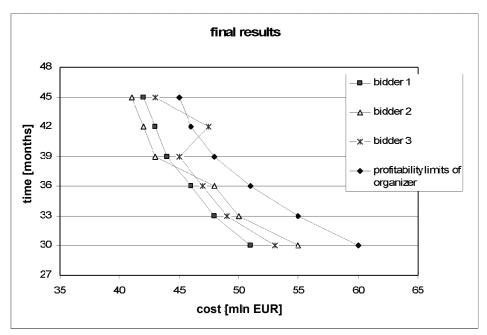


Figure 6. Offers in the final, fourth round

In the session presented here, we have observed that final offers tend to converge to the level of the second minimal profitability limit of the bidders. As we can see in Figure 5, the profitability limits of bidder 2 are lowest for the time variants 45, 42 and 39 months. Bidder 1 has the second minimal profitability limits for all the time variants. The profitability limits of bidder 3 are the second minimal ones for the time variants 36, 33 and 30 months. Let us compare the re-

sults of the final session presented in Figure 6. The winning offers of bidder 2 are on the level of the profitability limits of bidder 1 for 45, 42 and 39 months, and the winning offers of bidder 1 are on the level of the profitability limits of bidder 3 for 36, 33 and 30 months. It is understandable that the bidder with the lowest profitability limit for the given time variant has no incentive to decrease his offer and other bidders cannot beat it. In general, a large number of rounds can be required to obtain such a result, especially if the bidders are allowed to make only a small decrease in offers in each round.

5. Final remarks

The paper deals with mechanisms of multicriteria auctions in the context of incentive compatible decisions.

We have done an assessment of the rules for defining improvements of offers in successive rounds, based on the domination relation defined in the criteria space of the organizer. The rules differ with respect to the range of possible offers that can be proposed by bidders, and to the development of the auction process. It seems reasonable to apply different rules at different stages of the auction process. For example, at the beginning of the auction, the organizer may not be fully aware of his preferences. Therefore, a rule that enables the bidders to propose a wider range of offers can be applied, though the progress of such an auction can be rather slow. In further stages of the auction another, rather narrower, rule speeding up the progress could be applied, by limiting the range of possible offers.

We have constructed a mathematical model of the iterative multicriteria closed-bidding auction. It includes the formulation of the optimization task and implements the reference point approach of the multicriteria analysis performed by the organizer. The multi-agent computer-based system has been built to support the submission of offers, multicriteria analysis performed by the auction organizer, simulation and analysis of competing bidders' behavior.

The computer-based system used in the experimental studies ensures the confidentiality of private information about the profitability limits of the bidders and the organizer. We have done an assessment of the results of sessions conducted with the use of the system. We have observed that generally bidders are encouraged in the auction to gradually reveal their private information. Analogously to the Vickrey auction (see Vickrey, 1961), the proposed offers tend to converge in the consecutive rounds to the second minimal profitability limits of the bidders. This can be explained by the fact that the noncompetitive bidders, who must compete with the others to their limits, are motivated in the consecutive rounds to propose offers that tend to their profitability limits.

Further research may include development of the model and redesign of the multi agent computer-based system. Different rules of the multicriteria auction, and different strategies of bidders in the auction may be analyzed. Full confidentiality of individual information has been assumed in the model already proposed. The confidentiality relates to cost limits and preferences of the organizer and bidders. It is interesting to see how the access of bidders to some selected information, for example to the information on the organizer's preferences, may impact the behavior of the bidders and their strategies during the auction process. The bidders, in the model presented here, supply to the system data about their cost limits as well as the offers proposed. However, the corresponding multicriteria analysis leading to the calculation of that data has to be made outside the system. An additional module supporting such analysis would be useful. The cost limits of the organizer and the bidders impose obvious reservation points in multicriteria analysis performed by each of them. The cost limits can be calculated using the BATNA (Best Alternative to Negotiation Agreement) concept analogously to Kruś (2002; 2008; 2011). The BATNA concept (see Fisher, Ury, 1981) is commonly used in international negotiation processes.

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Dominik Kudyba*

ENERGY HEDGING USING GOAL PROGRAMMING

Abstract

Energy volume hedging is nowadays very important due to the current structure of the Polish energy market. Energy buyers plan their future demand, but its structure is very heterogeneous. In most cases, energy sellers can hedge energy by purchasing highly homogeneous futures contracts; thus some part of planned demand can't be effectively hedged. Unhedged open position should be minimized because of unknown cost at time *t*.

The aim of this paper is to propose a model of hedging electricity demand volume which minimizes open position. The problem has been solved using goal programming.

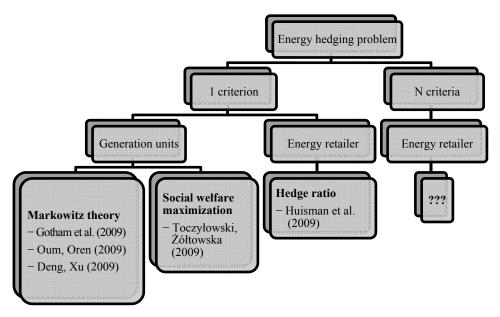
Keywords: energy hedging, goal programming, open position.

1. Introduction

The electricity market in Poland has been liberalized from the end of 1990s. This process gradually involves more areas of the electricity market. Usually, the TPA policy is the first impulse to changes. This policy strongly increases opportunities for competitions in every market with natural monopoly; without a doubt electricity infrastructure facilities are a classical example of natural monopoly in the theory of economics. Due to the TPA policy regarding the electricity market, every owner of an infrastructure facility has to grant access to the facility to every competitor who wants to use it. This means we can buy electricity from every firm offering it, not only from the one with infrastructure facilities close to our residence. The TPA policy changes the perception of electricity, which has now become a consumer good. The consumer can choose the best offer for him; hence, energy retailers have to compete.

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Many papers concerning electricity hedging problem exist in the literature. Most of them focus on optimization in generation units as a single criterion problem. For example, Gotham et al. (2009, p. 249-256) uses the Markowitz theory for optimizing energy production in generation units. However, Oum and Oren (2009, p. 43-56) as well as Deng and Xu (2009, p. 1523-1529) modify the standard mean-variance approach by using VaR as a risk measure. Toczyłowski and Żółtowska (2009) propose the maximization of social welfare coefficient for generation units. On the other hand Huisman et al. (2009, p. 169-174) deal with the hedging problem for energy retailers. There seem to be no papers dealing with the energy hedging problem for retailers as a multiple criteria problem. Schema 1 shows the results of recent literature studies.



Schema 1. Results of recent literature studies

The aim of this paper is to propose a model for fitting an offer which is satisfying for both the electricity consumer and retailer. Two criteria are considered: open position volume minimization (important for the retailer) and offer price minimization (important for the consumer).

The paper is divided into four parts. The first part contains basic definitions related to the problem of electricity hedging in energy market. The second part presents detailed information on the idea of electricity hedging and the formulation of the problem. After the problem formulation an illustrative example with results is given and, finally, conclusions with remarks on applying goal programming to the electricity hedging problem are presented.

2. Definitions

In general, an **energy consumer** is everyone using electricity: factories, enterprises, institutions such as schools, hospitals etc. and also households. In this paper, an energy consumer is a large production company with relatively high energy consumption level for whom electricity purchasing costs constitute a large share of its total expenses. Furthermore, the consumer has to be able to create an hourly schedule with **planned consumption** and to make it available to potential sellers. An example of a planned consumption schedule for the hours from 5:00 am to 4:00 pm for the first seven days of January 2013 is shown in Table 1.

Example of a planned consumption schedule

Table 1

Planned consumption [MW]												
Day/hour		5-6	6-7	7-8	8-9	9-10	10-11	11-12	12-13	13-14	14-15	15-16
2013-01-01	Mon	5,2	5,8	5,8	5,4	5,4	5,3	5,2	5,8	5,3	5	5,7
2013-01-02	Tue	5,6	5,5	5,6	5,3	5,6	5,9	6	5,9	5,4	5,8	5,9
2013-01-03	Wed	5,1	5,5	7,7	7,7	7,7	7,9	7,6	7,3	7,6	7,9	7,6
2013-01-04	Thu	5,8	5,2	7,3	8	7,4	7,6	7,7	7,5	7	7,6	7,9
2013-01-05	Fri	5,2	5,5	7,4	7,6	7,5	7,1	7,7	7,3	7,1	7,3	7,6
2013-01-06	Sat	5,1	5,8	5,6	6	6	5,8	5,5	5,4	5	5,5	5,5
2013-01-07	Sun	5,1	5,2	7,7	7,6	7,6	7,7	7,5	7,3	7,7	8	7,6

Source: Own elaboration.

We also consider an **energy retailer**. The basic difference between an energy retailer and a **generation unit** is that an energy retailer does not produce electricity. When the retailer is about to sell energy to a customer, he has to buy it first on the market. In other words, he has to **hedge** the client's volume on the market. We also assume that the **market** is the Polish Power Exchange¹. Schema 2 shows the relations between consumer, retailer, generation unit and market.



Schema 2. Relations between the electricity market participants

Polish Energy Exchange website: http://www.polpx.pl/en [23.03.2013].

There are two basic energy segments in the Polish Energy Exchange (POLPX)²:

- Day-Ahead Market (DAM),
- Commodity Forward Instruments Market with Physical Delivery (CFIM).

The Day-Ahead market consists of 24-hour markets. We can buy or sell energy for the individual hours for one day ahead (day t+1). This means that today we can buy energy only for tomorrow. In the Commodity Forward Market, on the other hand, electricity forward contracts with physical delivery are used. The basic difference is that on day t we can buy or sell energy for a week, month, quarter and even a year ahead. The price for the total energy delivered is fixed, so there is no price risk. The most liquid products are forward contracts for baseload and peakload. According to POLPX, a forward contract for baseload delivers equal amounts of energy in each hour of the supply period³. A forward contract for peakload includes supply in working days only for the hours from 7:00 am to $10:00 \text{ pm}^4$. Figure 1 presents a graphical illustration of the baseload and peakload delivery structure on a particular day, while Table 2 presents a supply matrix.

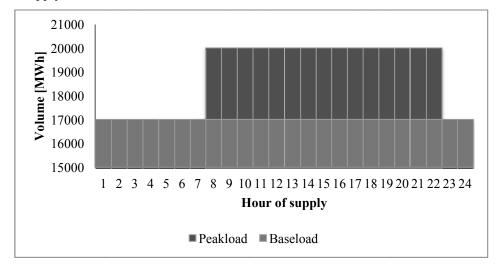


Figure 1. Baseload and peakload delivery structure

Source: Own elaboration using POLPX products definition.

² For more information about POLPX visit: http://www.polpx.pl/en [23.03.2013].

http://www.polpx.pl/en/42/commodity-forward-instruments-market-with-physical-delivery-cfim [23.03.2013].

Polish Power Exchange website: http://www.polpx.pl/en/42/commodity-forward-instruments-market-with-physical-delivery-cfim [23.03.2013].

Example of supply in baseload and peakload schedule												
Schedule of supply [MW]												
Day/hour 5-6 6-7 7-8 8-9 9-10 10-11 11-12 12-13 13-14 14-15 15-16												
2013-01-01	Mon	5	5	5	5	5	5	5	5	5	5	5
2013-01-02	Tue	5	5	10	10	10	10	10	10	10	10	10
2013-01-03	Wed	5	5	10	10	10	10	10	10	10	10	10
2013-01-04	Thu	5	5	10	10	10	10	10	10	10	10	10
2013-01-05	Fri	5	5	10	10	10	10	10	10	10	10	10
2013-01-06	Sat	5	5	5	5	5	5	5	5	5	5	5
2012 01 07	Sun	5	-	-	5	5	5	5	5	5	5	5

Table 2

Example of supply in baseload and peakload schedule

Source: Own elaboration using POLPX products definition.

3. The idea of hedging

The production company intending to buy electricity provides a schedule of future consumption to all interested retailers. The retailers present their offers as a feedback and the production company chooses the best offer. The retailers have to analyze the future consumption schedule for fitting in a structure of hedge. In general, hedge can be done in the DAM or in the CFIM segment. On the one hand, there is the highly heterogeneous structure of planned consumption (see Table 1), and there is also the very homogeneous structure of supply of market products (Table 2). Some surpluses and shortages occur in particular hours – these differences are called the open position volume.

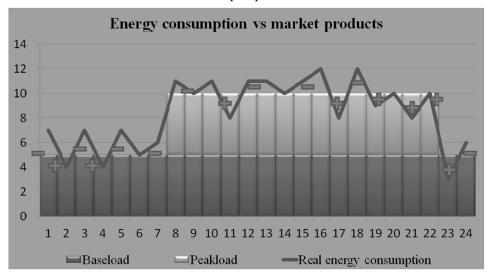


Figure 2. Consumption schedule and forward contracts

Source: Own elaboration.

The problem is to fit in a hedge as a combination of volume bought in the DAM and CFIM segments taking into account both market segments. The retailer is trying to hedge as much as possible in forward contracts because it is possible to hedge volumes for all supply periods at once. And, most importantly, the price of hedge is fixed and therefore the price risk is smaller. Hence, it is most important for the retailer to minimize the open position volume, while the consumer is interested only in the low price of the offer⁵. Concluding, the hedging problem can be formulated as a multicriteria optimization problem which minimizes the open position volume and offer price.

$$\begin{split} & \underset{x_{base}}{\textit{MIN}} \left\{ \sum_{i} \sum_{j} \left| \left(x_{base} \cdot b_{ij} + x_{peak} \cdot p_{ij} \right) - z_{ij} \right| \right\} \\ & \underset{x_{base}}{\textit{MIN}} \left\{ \sum_{i} \sum_{j} \left[\frac{c_{base} \cdot x_{base} \cdot b_{ij} + c_{peak} \cdot x_{peak} \cdot p_{ij}}{x_{base} \cdot b_{ij} + x_{peak} \cdot p_{ij}} \right] \right\} \\ & \text{where:} \\ & \left\{ x_{base} \leq \max_{i,j} \left\{ \left(b_{ij} - p_{ij} \right) z_{ij} \right\} \right. \\ & \left. x_{base} \geq \min_{i,j} \left\{ \left(b_{ij} - p_{ij} \right) z_{ij} \right\} \right. \\ & \left. x_{peak} \leq \max_{i,j} \left\{ p_{ij} z_{ij} \right\} \right. \\ & \left. x_{peak} \geq \min_{i,j} \left\{ p_{ij} z_{ij} \right\} \right. \\ & \left. x_{base}, x_{peak} \in Int \right. \\ & i = 1, \dots, 365 \\ & j = 1, \dots, 24 \end{split}$$

Decision variables:

 x_{base} – baseload power, x_{peak} – peakload power.

Available data:

 $Z_{i \times j}$ – future consumption schedule, c_{base} – forward's baseload price, c_{peak} – forward's peakload price, i – day index, for example i= 1, ..., 365 for supply in year 2013, j – hour index in i-th day, j= 1, ..., 24, $B_{i \times j}$ – baseload supply matrix, $P_{i \times j}$ – peakload supply matrix.

⁵ Energy retailer is fully responsible for every aspect of energy supply. That's why the consumer doesn't care about the volume – he's interested only in the price of energy.

Supply matrices are binary matrices. When there is supply in the *i-th* day and the *j-th* hour, the value is 1, otherwise the value is 0. Due to the definition of supply in baseload, the matrix $B_{i\times j}$ is filled with 1s. The matrix $P_{i\times j}$ is filled with 1s in the intersections of the rows representing the hours from 7 to 22 and working days. Criterion (1) is responsible for minimizing the open position volume as an absolute value: we want to minimize surpluses as well as shortages in the open position volume. Criterion (2) minimizes the volume weighed average price of forward contracts in hedge⁶. Constraint (3) defines the search range for the baseload power forward contract. Constraint (4) defines the search range for the peakload power forward contract.

We want to minimize surpluses as well as shortages in the open position volume; the price should be less than or equal to some specified upper level P_u . We can transform this basic model into a goal programming model⁷.

$$\begin{split} & \underset{x_{base}, x_{peak}}{MIN} \quad \{d_1^+ + d_1^- + d_2^+\} \\ & \underset{d_1^+, d_1^-, d_2^+}{d_1^+, d_1^-, d_2^+} \\ & \left(\sum_{i} \sum_{j} \left(x_{base} \cdot b_{ij} + x_{peak} \cdot p_{ij} - z_{ij} \right) - d_1^+ + d_1^- = 0 \right. \\ & \left(\sum_{i} \sum_{j} \left(\frac{c_{base} \cdot x_{base} \cdot b_{ij} + c_{peak} \cdot x_{peak} \cdot p_{ij}}{x_{base} \cdot b_{ij} + x_{peak} \cdot p_{ij}} \right) - d_2^+ + d_2^- = P_u \right. \\ & \left. x_{base} \leq \max_{i,j} \left\{ \left(b_{ij} - p_{ij} \right) z_{ij} \right\} \right. \\ & \left. x_{base} \geq \min_{i,j} \left\{ \left(b_{ij} - p_{ij} \right) z_{ij} \right\} \right. \\ & \left. x_{peak} \leq \max_{i,j} \left\{ p_{ij} z_{ij} \right\} \right. \\ & \left. x_{peak} \geq \min_{i,j} \left\{ p_{ij} z_{ij} \right\} \right. \\ & \left. x_{base}, x_{peak} \in \text{Int} \right. \\ & \left. d_1^+, d_1^-, d_2^+, d_2^- \geq 0 \right. \\ & i = 1, \dots, 365 \\ & j = 1, \dots, 24 \end{split}$$

where:

 d_1^+, d_1^- – overachieve and underachieve coefficients, respectively, for the open position volume,

 d_2^+ , d_2^- overachieve and underachieve coefficients, respectively, for the offer price, P_u – fixed upper price level specified by the decision maker.

We assume zero sales margin, so that offer price is equal to purchasing costs.

According to: Trzaskalik (red.) (2006, p. 39-40).

The advantage is that there is only one criterion consisting of overachieve and underachieve coefficients. Furthermore, we want to minimize only the surpluses of price criterion, so the variable d_2^- does not occur in the objective function. There is no need to employ sophisticated routines for nonsmooth functions⁸.

4. Illustrative example

Let us consider a future consumption schedule for the first seven days of January 2013. The potential retailer has to hedge 90 GWh of energy. Consumption is higher in peak hours (see Figure 3). The average baseload power is 508 MW and the average peak power is 107 MW, for the total of 615 MW consumption in peak hours.

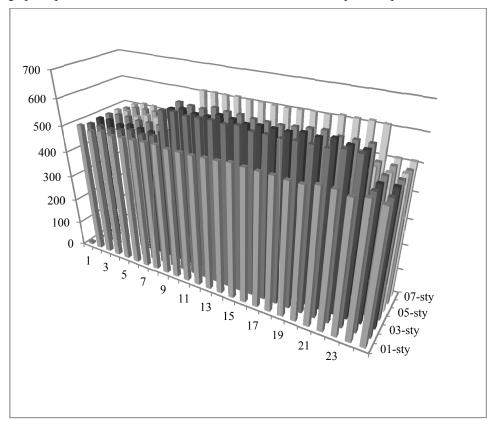


Figure 3. Graphical representation of consumption structure

Source: Own elaboration.

The first criterion in the basic model includes absolute value – this makes it nonsmooth.

The forward contract price is 197,00 PLN/MWh for baseload and 213,00 PLN/MWh for peakload. We also assume that the upper price level P_u is equal to the peakload price of 213,00 PLN/MWh – we want the price of our hedge to be less than or equal to 213,00 PLN/MWh.

The results are presented in Table 3. One solution has been obtained by using the goal programming model (marked in bold). But offering to the potential customer one offer (solution) is not enough – there are no negotiation possibilities with only one offer. Therefore three additional solutions have been generated, which are very interesting because of their price. In the goal programming solution the open position volume is the least but the price is the largest; the consumer would probably not choose this offer. But the retailer should consider additional solutions, in which the open position volume is only 0,06%-0,20% higher and whose price is more attractive for the potential client.

Table 3
The results

Solution	OP [MWh]	Price [PLN/MWh]	Base [MW]	Peak [MW]	OP/Volume
Goal programming	0,68	199,87	504	110	0,00%
Additional	-59,32	199,85	504	109	0,06%
Additional	-119,32	199,82	504	108	0,13%
Additional	-179,32	199,80	504	107	0,20%

Source: Own elaboration.

5. Summary

Goal programming has been used to solve the electricity hedging problem consisting of two different criteria: open position volume and offer price. This multicriteria problem is more elastic than other such problems in the literature. The advantage is that both the retailer and the consumer can use it to calculate the parameters of a reference offer for the negotiation process. Another advantage is that the retailer has an effective tool for decision making during the negotiation process with which he can generate solutions including potential changes in the parameters of the model. The decision maker makes a decision about the trade-off between the open position volume and the price of the offer.

Goal programming is an effective way to simplify the basic energy hedging problem. Being a very convenient tool, it makes this model applicable in practice. Goal programming is also a very ingenious method for handling nonsmooth criteria indirectly. The goal programming approach has one important disadvantage: it generates only one solution at a time (in our paper additional solutions have been generated). Other methods provide a set of nondominated solutions which may be more useful for the decision maker.

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IMPROVING THE GAME APPROACH TO FUZZY MADM

Abstract

In the FSS paper 157 (2005, p. 34-51) we presented a game approach for solving MADM problems with fuzzy decision matrix. The results of the paper essentially depend on the assumption that the entries of the fuzzy decision matrix are triangular fuzzy numbers and dependent via a real parameter λ . In this paper we present a more general game approach for solving fuzzy MADM problems free of these restrictions. The entries of the decision matrix are assumed to be not necessarily dependent fuzzy intervals with bounded support as defined by Dubois and Prade.

Keywords: Fuzzy MADM, Fuzzy interval, game against Nature, Nash equilibrium.

1. Introduction

In traditional Multiple Attribute Decision Making (MADM) problems it is assumed that the evaluations of alternatives with respect to attributes are known exactly by the decision maker (DM) (Hwang, Yun, 1981). This restriction limits the scope of real-world application of the traditional approaches. Indeed, it often happens that the DM doesn't know exactly the evaluations of the alternatives with respect to attributes. This situation occurs when the DM is uncertain about the behavior of the environment. The uncertainty in evaluations may be of dif-

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ferent types: probabilistic, fuzzy, fuzzy-probabilistic, etc. In this paper we deal with uncertainty of fuzzy type. When fuzzy uncertainty is involved, we say that the DM faces a fuzzy MADM problem. The most adequate tool to handle such type of problems is the fuzzy set theory introduced by Zadeh (1965). Several approaches have been developed for solving fuzzy MADM problems. We can classify them into two classes. The first class consists of methods that use different ways of ranking fuzzy numbers; for each alternative a fuzzy score is calculated, then the best alternative is selected based on the ranking method used. The second one is based on different ordering of fuzzy numbers. In Chen, Hwang (1992), the most important methods for solving fuzzy MADM problems are described. In our paper (Chen, Larbani, 2005), we have introduced a new approach for solving a fuzzy MADM problem by transforming it into a game against Nature, via α -cuts and maxmin criterion of decision making under uncertainty (Chen, Larbani, 2005; Larbani, 2009a; Larbani, 2009b). And our work inspired several papers dealing with the fuzzy game approach for MADM later; for example, see the papers by Kahraman (2008), Larbani (2009a; 2009b), Clemente et al. (2011), Yang and Wang (2012), etc. The results of the paper essentially depend on the assumption that the entries of the fuzzy decision matrix are triangular fuzzy numbers and are dependent via a real parameter λ . In this paper we present a more general game approach for solving fuzzy MADM problems free of these two restrictions. Indeed, in this approach, unlike in Chen, Larbani (2005), the entries of the decision matrix are assumed to be fuzzy intervals with bounded support as defined by Dubois and Prade (2000) and not necessarily dependent. Thus, the scope of application to real-world problems will be much larger than the one of the approach developed in Chen, Larbani (2005). As in Chen, Larbani (2005), in this paper, we also formulate the fuzzy MADM problem as a two-person zero-sum game against Nature with an uncertain payoff matrix via α -cuts and maxmin principle. However, the game we obtain and the solution we propose and its computation method are totally different from those developed in Chen, Larbani (2005).

The paper is organized as follows. In section 2, we present the fuzzy MADM problem. In Section 3, we present our method step by step. Then we provide a procedure for computation of the solution we propose. In section 4, we illustrate the method by an application. Section 5 concludes the paper.

2. Problem Statement

Let us consider an MADM problem with the following fuzzy decision matrix:

$$C_{1} \qquad C_{2} \quad \dots \quad C_{n}$$

$$A_{1} \begin{bmatrix} \widetilde{a}_{11} & \widetilde{a}_{12} & \dots & \widetilde{a}_{1n} \\ \widetilde{a}_{21} & \widetilde{a}_{22} & \dots & \widetilde{a}_{2n} \\ \dots & & & \\ A_{m} \begin{bmatrix} \widetilde{a}_{m1} & \widetilde{a}_{m2} & \dots & \widetilde{a}_{mn} \end{bmatrix}$$

$$(1)$$

where m alternatives A_i , i = 1,2,...,m are evaluated with respect to n attributes C_j , j = 1,2,...,n; \widetilde{a}_{ij} represents the evaluation of alternative i with respect to attribute j. The objective of the decision maker (DM) is to select the *best* alternative according to the available information in the fuzzy matrix (1). Let us recall the definition of a fuzzy interval with bounded support as defined by Dubois and Prade (2000).

Definition 2.1 (Dubois, Prade, 2000). A fuzzy interval \widetilde{F} with bounded support is defined by $\widetilde{F} = (R, \mu(.))$ with $\mu_{\widetilde{F}}(.): R \to [0,1]$ satisfying the following conditions:

- (i) $\mu_{\widetilde{F}}(x) = 0$ for all $x \in (-\infty, c]$,
- (ii) $\mu_{\widetilde{F}}(.)$ is right-continuous non-decreasing on [c,a],
- (iii) $\mu_{\widetilde{F}}(x) = 1$ for all $x \in [a,b]$,
- (iv) $\mu_{\widetilde{E}}(.)$ is left-continuous non-increasing on [b,d],
- (v) $\mu_{\widetilde{E}}(x) = 0$ for all $x \in [d, +\infty)$,

where $-\infty < c \le a \le b \le d < +\infty$, and R is the real line.

We say that a fuzzy interval with bounded support $\widetilde{F} = (R, \mu_{\widetilde{F}}(x))$ is positive if its support satisfies:

$$Sup(\widetilde{F}) = \{ z \mid z \in R, \ \mu_{\widetilde{F}}(z) > 0 \} \subset [0,+\infty).$$

We make the following assumption.

Assumption 2.1. The DM assumes that the entries of \widetilde{D} are positive fuzzy intervals as defined by Dubois and Prade.

Thus, we obtain an MADM problem with a fuzzy decision matrix under Assumption 2.1.

Remark 2.1. It is important to note that the fuzzy MADM problem (1) under Assumption 2.1 is more general than the fuzzy MADM treated in Chen, Larbani (2005). Indeed, in Chen, Larbani (2005) the entries of the fuzzy decision matrix are assumed to be triangular fuzzy numbers and dependent via a real parameter λ . In the fuzzy MADM problem (1) the entries of the fuzzy matrix are not assumed to be dependent and belong to the class of fuzzy intervals with bounded support as defined by Dubois and Prade (2000), which is more general than the class of triangular fuzzy numbers. Thus, the class of MADM problems that can be solved using the model (1) is much larger than the class of MADM problems that can be solved using the model in Chen, Larbani (2005).

3. The Method

In this section we present our approach and the resolution procedure. We transform the initial fuzzy MADM problem into a two-person zero-sum game between the DM and Nature. Then based on the solution of this game, we provide a procedure for selecting the best alternative. As in Chen, Larbani (2005), this game is obtained via α -cuts and maxmin principle of decision making under uncertainty. The use of α -cuts is based on the approach of Sakawa and Yano (1989) for solving multiobjective non linear problems with fuzzy parameters. In addition to the differences we have mentioned in Remark 2.1, the game we obtain in this paper and the resolution procedure are totally different compared to those of Chen, Larbani (2005). We present the method in four steps. We start by constructing the α -cuts of the entries of the fuzzy decision matrix \widetilde{D} of the problem (1). In the second step, we introduce the game against Nature. In the third step we solve the game obtained in the second step. Finally, we propose a procedure for the selection of the best alternative.

3.1. Defuzzification

Suppose that the DM has chosen an α -cut level α . Then, following the approach of Sakawa and Yano (1989), for each entry \widetilde{a}_{ij} of the fuzzy decision matrix \widetilde{D} , we obtain the α -cut:

$$[\widetilde{a}_{ij}]^{\alpha} = \{ a_{ij} \mid \mu_{\widetilde{a}_{ij}}(a_{ij}) \ge \alpha \}, \ i = \overline{1, m} \text{ and } j = \overline{1, n}$$
 (2)

In our model we interpret confidence as "degree of certainty of truth", then an α -cut level can be interpreted as a degree of necessity (Dubois, Prade, 2000). We assume that once the DM has chosen the level α , then he is certain (with degree of necessity 1) that for each alternative i and attribute j, the evaluation of i with respect to j is in the α -cut $[\widetilde{a}_{ij}]^{\alpha}$, but he doesn't know which particular $a_{ij} \in [\widetilde{a}_{ij}]^{\alpha}$ is the ac-

tual evaluation of the alternative i with respect to attribute j. Hence, the decision maker faces a MADM problem with crisp uncertain evaluations that vary in the α -cuts (2). This problem can be represented as follows:

$$C_{1} C_{2} \dots C_{n}$$

$$A_{1} \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

$$(3)$$

where each entry a_{ij} is a crisp parameter that can take any value in the α -cut $[\widetilde{a}_{ij}]^{\alpha}$, for $i = \overline{1,m}$ and $j = \overline{1,n}$. Such a problem is known in literature as the decision making problem under uncertainty in the case of complete ignorance (Luce, Raiffa, 1957; Nash, 1951; Rosen, 1965). In the next section we introduce a game approach to solve it.

3.2. The Game and the Selection of the Best Alternative

Since the problem (3) is a special decision making problem under uncertainty in the case of complete ignorance, we can use one of the criteria of decision making under uncertainty to solve it Luce, Raiffa (1957). In this paper we assume that the decision maker is conservative with respect to the possible realizations of the unknown parameters (evaluations) a_{ij} in $\left[\widetilde{a}_{ij}\right]^{\alpha}$, for $i=\overline{1,m}$ and $j=\overline{1,n}$. Then the most adequate criterion to use is the maxmin (Wald) criterion. Consequently, the problem can be treated as a game against Nature. In this game the DM wants to maximize his payoff and Nature wants to minimize the same payoff. The DM chooses the alternatives A_i , i=1,2,...,m; Nature chooses the evaluations a_{ij} , $i=\overline{1,m}$ and $j=\overline{1,n}$, i.e. the entries of the matrix D. Here the DM considers Nature as an "intelligent player" who wants to minimize his payoff. Formally, this crisp zero-sum two person game can be represented as follows:

$$G_2 = (S^m, \prod_{\stackrel{1 \le i \le m}{1 \le j \le n}} [\widetilde{\mathbf{a}}_{ij}]^{\alpha}, \ N(x, a))$$

$$\tag{4}$$

where $S^m = \{x = (x_1, x_2, ..., x_m), x_i \ge 0, i = \overline{1, m}, \sum_{1}^{m} x_i = 1\}$, is the set of mixed strategies of the DM; Nature chooses the entries a_{ij} for $i = \overline{1, m}$ and $j = \overline{1, n}$ of the matrix D in the set $\prod_{1 \le i \le m \atop 1 \le j \le m} [\widetilde{a}_{ij}]^{\alpha}$.

The payoff function of the decision maker is $N(x,a) = \sum_{\substack{i=1...m\\i=1}} x_i a_{ij}$, where

$$a = (a_{ij})_{\substack{1 \le i \le m \\ 1 \le j \le n}} \in \prod_{\substack{1 \le i \le m \\ 1 \le j \le n}} [\widetilde{\mathbf{a}}_{ij}]^{\alpha}$$
 and $x \in S^m$, the payoff of Nature is just the negative

of the DM's payoff i.e. -N(x,a). We justify the definition of the payoff function of the DM as follows. Once the DM and Nature have chosen their strategies $x = (x_1, x_2, ..., x_m) \in S^m$ and $a = (a_{ij})_{\substack{1 \le i \le m \\ 1 \le j \le n}} \in \prod_{i \le j \le m \atop 1 \le j \le n} [\widetilde{a}_{ij}]^{\alpha}$. The payoff of the DM with

respect to any alternative *i* can be naturally defined as:

$$x_i \sum_{i=1}^n a_{ij} \tag{5}$$

Indeed, x_i is the probability (or weight) that he assigns to the alternative i and the

sum
$$\sum_{j=1}^{n} a_{ij}$$
 is just the aggregated score of the alternative *i* with respect to all the *n*

attributes if it was chosen with probability $x_i = 1$. Then the overall payoff of the DM can be rationally defined as the sum of the payoffs with respect to all the alternatives i.e.:

$$\sum_{i=1}^{m} x_i \sum_{i=1}^{n} a_{ij} = \sum_{i=1}^{m} \sum_{i=1}^{n} a_{ij} x_i = N(x,a)$$

On the other hand, x_i can also be interpreted as the proportion of times the DM should selects the alternative i as best alternative if the decision making problem is repeated a certain number of times.

In the next section we deal with the problem of resolution of the game (4). Now based on Nash equilibrium of the game (4), we propose the following definition of the best alternative for the DM.

Definition 3.1. Assume that (x^0, a^0) is a Nash equilibrium (Luce, Raiffa, 1957; Nash, 1951; Rosen, 1965) of the game (4), then the best alternative for the DM is defined as the alternative that has the maximum score, that is, it is the alternative i_0 that satisfies:

$$\max_{1 \le i \le m} \{ x_i^0 \sum_{j=1}^n a_{ij}^0 \} = x_{i_0}^0 \sum_{j=1}^n a_{i_0 j}^0$$
 (6)

We call it α – maxmin best alternative.

Remark 3.1. Note that in the definition (5) of the score of an alternative i, we assume that the DM considers the attributes equally important. If the DM wants to assign different positive weights w_j , j = 1,..n to attributes C_j , j = 1,..n respectively, then the score of any alternative i can be defined as follows:

$$x_i \sum_{j=1}^n w_j a_{ij} \tag{7}$$

The results of this paper are also valid if the score (7) is used instead of the score (5). In the sequel of the paper, unless specified, for simplicity of the presentation, we will assume that the weights assigned by the DM are $w_i = 1$, j = 1,...n, that is, we will use the score (5) for alternatives.

3.3. Resolution of the Game

In this section we study the problem of existence of a solution to the game (4) and its computation. Note that the game (4) is not a traditional matrix twoperson zero-sum game, because Nature chooses the entries a_{ij} for i = 1, m and $j = \overline{1,n}$ of the matrix D in the set $\prod [\widetilde{a}_{ij}]^{\alpha}$. The game (4) is an infinite two-

person zero-sum game with variable payoff matrix. Consequently, the existence of Nash equilibrium is not guaranteed. Thus, we first deal with the problem of the existence of Nash equilibrium of the game (4), then address the problem of its computation.

Proposition 3.1. The game (4) has a Nash equilibrium.

Proof. By definition, the set S^m is convex and compact. Since the entries \tilde{a}_{ii} for $i = \overline{1,m}$ and $j = \overline{1,n}$ of the fuzzy decision matrix in the problem (1) are fuzzy intervals with bounded support as defined by Dubois and Prade (2000), the α -cuts $[\widetilde{a}_{ij}]^{\alpha}$, for $i = \overline{1, m}$ and $j = \overline{1, n}$ are closed intervals in the real line, hence they are convex and compact sets. The function $x \to N(x, a) = \sum_{\substack{i=1..m\\j=1..n}} x_i a_{ij}$

is linear for all $a \in \prod_{1 \le i \le m \atop 1 \le j \le n} [\widetilde{a}_{ij}]^{\alpha}$, hence it is concave on S^m , for all $a \in \prod_{1 \le i \le m \atop 1 \le j \le n} [\widetilde{a}_{ij}]^{\alpha}$. The function $a \to -N(x,a) = -\sum_{\substack{i=1..m \\ j=1..n}} x_i a_{ij}$, is also linear for all $x \in S^m$, hence it is

concave on $\prod [\widetilde{a}_{ij}]^{\alpha}$, for all $x \in S^m$. From the foregoing we deduce that all the

conditions of the theorem of the existence of Nash equilibrium (Luce, Raiffa, 1957; Nash, 1951; Rosen, 1965) are satisfied by the game (4). Thus, it has a Nash equilibrium.

Let us recall that a strategy profile (x^0, a^0) is a Nash equilibrium of the game (4) if x^0 is a best response of DM to the strategy a^0 of Nature, and a^0 is a best response of Nature to the strategy x^0 of DM, that is:

$$\max_{x \in S^{m}} N(x, a^{0}) = N(x^{0}, a^{0}) \text{ and } \max_{a \in \prod_{1 \le i \le m}} -N(x, a^{0}) = -N(x^{0}, a^{0})$$

In the following proposition we show how a Nash equilibrium of the game (4) can be computed.

Proposition 3.2. Let $[\widetilde{a}_{ij}]^{\alpha} = [(a_{ij}^{\alpha})^{L}, (a_{ij}^{\alpha})^{U}]$, for all $i = \overline{1, m}$ and $j = \overline{1, n}$. Then the pair (x^{α}, a^{α}) where $a^{\alpha} = (a_{ij}^{\alpha})_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n}}^{L}$ and x^{α} is an optimal solution to the

linear programming problem:

$$Max N(x, a^{\alpha}) = \sum_{\substack{i=1..m\\j=1..n}} x_i (a_{ij}^{\alpha})^L,$$

$$x \in S^m$$
(8)

is a Nash equilibrium of the game (4).

Proof. Let us prove that the strategy of Nature $a^{\alpha} = (a_{ij}^{\alpha})_{\substack{1 \le i \le m \\ 1 \le j \le n}}^{L}$ is the best re-

sponse to any strategy $\overline{x} \in S^m$ of the DM. Indeed, for any $x \in S^m$, $x_i \ge 0$, $i = \overline{1,m}$, then $-N(\overline{x},a) = -\sum_{\substack{i=1..m\\j=1..n}} \overline{x}_i a_{ij} \le -\sum_{\substack{i=1..m\\j=1..n}} \overline{x}_i \left(a_{ij}^{\alpha}\right)^L = -N(\overline{x},a^{\alpha})$, for all

$$a \in \prod_{1 \le i \le m \atop 1 \le i \le m} [\widetilde{a}_{ij}]^{\alpha}$$
. In particular, for $\overline{x} = x^{\alpha}$, we get $-N(x^{\alpha}, a) \le -N(x^{\alpha}, a^{\alpha})$, for all

 $a \in \prod_{1 \le i \le m \atop 1 \le j \le n} [\widetilde{a}_{ij}]^{\alpha}$. On the other hand, since x^{α} is an optimal solution to the problem (8),

it is a best response to the strategy $a^{\alpha} = (a_{ij}^{\alpha})_{\substack{1 \le i \le m \\ 1 \le j \le n}}^{L}$ of Nature. Thus, (x^{α}, a^{α}) is a Nash equilibrium of the game (4) (Luce, Raiffa, 1957).

3.4. Procedure for Selecting the Best Alternative

In this section we provide a procedure for selecting the best alternative. Moreover, the alternatives can also be ranked from the best to the worst.

Procedure 3.1

Step 1. Ask the DM to provide the α -cut level, then compute the α -cuts $[\widetilde{a}_{ij}]^{\alpha} = [(a_{ij}^{\alpha})^{L}, (a_{ij}^{\alpha})^{U}], i = \overline{1, m} \text{ and } j = \overline{1, n}.$

Step 2. Solve the linear programming problem (8) with $a^{\alpha} = (a_{ij}^{\alpha})_{\substack{1 \le i \le m \\ 1 \le j \le n}}^{L}$. Let x^{α} be an optimal solution of (8), then (x^{α}, a^{α}) is a Nash equilibrium of the game (4).

Step 3. For each alternative *i* calculate its individual score $x_i^{\alpha} \sum_{j=1}^n a_{ij}^{\alpha}$. Then

rank the alternatives based on their score, the best being the one with the largest score.

We illustrate Procedure 3.1 by Example 3.1 below.

Remark 3.2. If the DM provides a specific level α_{ij} for each alternative i and attribute j, $i = \overline{1,m}$ and $j = \overline{1,n}$, in Step 1 of Procedure 3.1, the value $\alpha = \max(\alpha_{ij})$ can be chosen as a common level. This choice can be justified by the fact that $\left[\widetilde{\alpha}_{ij}\right]^{\alpha} \subset \left[\widetilde{\alpha}_{ij}\right]^{\alpha_{ij}}$, for all $i = \overline{1,m}$ and $j = \overline{1,n}$.

3.5. A more General Model

In this section we assume that in order to face the uncertainty in evaluations, the DM chooses not only the mixed strategy $x \in S^m$ but the attribute weights w_j , j = 1,..n as well (7). Using the same approach, we obtain the following extension of the game (4):

$$G_2 = (S^m \times S^n, \prod_{1 \le i \le m \atop 1 \le i \le dn} [\widetilde{\mathbf{a}}_{ij}]^{\alpha}, \ N((x, w), a))$$

$$(9)$$

where $S^n = \{ w = (w_1, w_2, ..., w_n), w_j \ge 0, j = \overline{1, n}, \sum_{j=1}^{n} w_j = 1 \}$, the strategies of the

DM are pairs
$$(x,w) \in S^m \times S^n$$
; the payoff of the DM is $N((x,w),a) = \sum_{\substack{i=1..m\\i=1\ n}} x_i w_j a_{ij}$

and the payoff of Nature is -N((x,w),a). We have the following definition.

Definition 3.2. Assume that $((x^0, w^0), a^0)$ is a Nash equilibrium of the game (9), then the best alternative for the DM is defined as the alternative that has the maximum score, that is, it is the alternative i_0 that satisfies:

$$\max_{1 \le i \le m} \{ x_i^0 \sum_{j=1}^n w_j^0 a_{ij}^0 \} = x_{i_0}^0 \sum_{j=1}^n w_j^0 a_{i_0 j}^0$$
 (10)

We call it α w – maxmin best alternative. We have the following proposition which is similar to Proposition 3.2.

Proposition 3.3. Let $[\widetilde{a}_{ij}]^{\alpha} = [(a_{ij}^{\alpha})^{L}, (a_{ij}^{\alpha})^{U}]$, for all $i = \overline{1, m}$ and $j = \overline{1, n}$. Then the pair (x^{α}, a^{α}) , where $a^{\alpha} = \prod_{i=1}^{n} (a_{ij}^{\alpha})^{L}$ and x^{α} is an optimal solution to the

linear programming problem:

Max N((x, w),
$$a^{\alpha}$$
) = $\sum_{\substack{i=1..m\\j=1..n}} x_i w_j a_{ij}$ (11)
 $x \in S^m, w \in S^n$

is a Nash equilibrium of the game (9). The proof is similar to that of Proposition 3.2. **Procedure 3.2**

Step 1. Ask the DM to provide the α -cuts level α , then determine $[\widetilde{a}_{ij}]^{\alpha} = [(a_{ij}^{\alpha})^{L}, (a_{ij}^{\alpha})^{U}]$, for all $i = \overline{1, m}$ and $j = \overline{1, n}$.

Step 2. Find a Nash equilibrium $((x^0, w^0), a^0)$ of the game (9) using Proposition 3.3.

Step 3. For each alternative *i* calculate its individual score $x_i^0 \sum_{j=1}^n w_j^0 a_{ij}^0$. Then rank the alternatives based on their score, the best being the one with the largest score

Remark 3.3. Let $A=\{A_i \mid x_i^\alpha=0\}$ be the set of alternatives with zero weight, and $\overline{A}=\{A_i \mid x_i^\alpha>0\}$ be the set of alternatives with positive weights. It may happen in Procedure 3.1 or 3.2 that $A\neq\varnothing$. In this case the implemented procedure divides the set of alternatives into two classes A and \overline{A} . The DM is indifferent regarding the alternatives in the class A, moreover they are the least alternatives. On the other hand, he can rank the alternatives in \overline{A} according to their scores. As an extreme case it may happen that for an alternative A_{i_0} , $x_{i_0}^\alpha=1$, then we have, $x_i^\alpha=0$, for all $i\neq i_0$, i.e. $A=\{A_i \mid i\neq i_0\}$ and $\overline{A}=\{i_0\}$. It clear that i_0 is, absolutely, the best decision for its score is better than the score of any other alternative. This case happens when the alternative i_0 dominates all the other alternatives for all $a\in\prod_{1\leq i\leq m}[\widetilde{a}_{ij}]^\alpha$ i.e $a_{i_0,j}^\alpha>a_{ij}^\alpha$, for all $i\neq i_0$.

Remark 3.4. From a computational point of view, the Procedures 3.1 and 3.2 are simpler than the procedure developed in Chen, Larbani (2005). In Procedures 3.1 and 3.2 one has to solve only one linear programming problem (8) and (11) respectively, while in the procedure of Chen, Larbani (2005) several linear programming problems have to be solved.

4. Example and Discussion

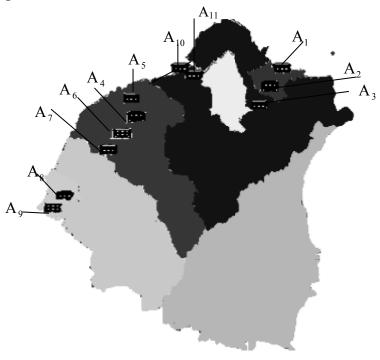


Figure 1. The Candidate Locations around the Taipei Metropolitan (Yellow District)

In this section, we will apply Procedure 3.1 to validate the presented game approach. The case study is describes a logistics company in Taiwan, which wants to select an appropriate location in the northern part of Taiwan for business expansion. Five experienced experts from various vendors and customers of this logistics company are invited to rank eleven candidate warehouse locations in Figure 1. The necessary attributes for appropriately ranking the location of warehouse are collected – these attributes are land cost (C_1) , labor cost (C_2) , degree of traffic congestion (C_3) , accessibility to the rapid transit system (C_4) , accessibility to the industrial park (C_5) , accessibility to the international airport (C_6) and accessibility to the international harbor (C_7) . These experienced logistics managers are asked to provide the evaluations of the locations with respect to the attributes. These fuzzy values are ranged within the quality interval from 1 to 10, where "1" means the lowest degree and "10" means the highest degree. When using the information in a decision matrix, the values of all attributes are normalized so that for all of them, beneficial or not, higher values are preferred. A matrix of type (1) with triangular fuzzy entries is obtained as a result (see appendix). The DM with the help of logistic managers fix the α -cut level α , then the α -cuts $[\widetilde{a}_{ij}]^{\alpha}$ are determined, and a decision matrix of type (3) is obtained. Next, a game of type (4) is solved. The computed scores of $x_i^{\alpha} \sum_{j=1}^{n} a_{ij}^{\alpha}$ with respect to various α -cut levels are shown in Table 1.

Table 1 Ranking Scores of the Eleven Candidate Locations

Location $\setminus \alpha$	0.1	0.6	0.8
A_1	0.00(2)	0.00(2)	0.00(2)
A_2	0.00(2)	0.00(2)	0.00(2)
A_3	0.00(2)	0.00(2)	0.00(2)
A_4	47.9 (1)	52.4 (1)	54.2 (1)
A_5	0.00(2)	0.00(2)	0.00(2)
A_6	0.00(2)	0.00(2)	0.00(2)
A_7	0.00(2)	0.00(2)	0.00(2)
A_8	0.00(2)	0.00(2)	0.00(2)
A_9	0.00(2)	0.00(2)	0.00(2)
A_{10}	0.00(2)	0.00(2)	0.00(2)
A_{11}	0.00(2)	0.00(2)	0.00(2)

Note: () denotes the rank.

For the three different levels of α , we obtained the same optimal strategy for the DM, $x_4^{\alpha}=1$ and $x_j^{\alpha}=0$, for $j \neq 4$. It is clear that Alternative 4 is the best choice because $x_4^{\alpha}=1$ and $x_j^{\alpha}=0$, for $j \neq 4$. The computed priority of each alternative is quite stable: as the α -cut level changes, the fuzzy score of alternatives varies but the priority of each alternative is still the same. The logistics practitioners were very satisfied with the simplicity, effectiveness and outcome of the proposed method. Note that in our approach the DM can choose different α -cut levels in order to check the sensitivity of the best solution with respect to the level α .

5. Conclusions

In this paper we have considerably improved the game approach to fuzzy MADM proposed in Chen, Larbani (2005). Compared to the approach in Chen, Larbani (2005), our approach is more general in the sense that it doesn't require the dependence of the evaluations of alternatives with respect to attributes and the fuzziness of these evaluations is of a more general type: fuzzy intervals with bounded support. Thus, this approach is capable of handling a wider class of

fuzzy MADM problems. We think that the game approach for solving fuzzy decision making problems is not well explored; more interesting results can be obtained in this direction of research.

Appendix

Fuzzy Decision Matrix for Location Decision

Alternatives/Attributes	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	5,6,7	7,8,9	5,6,7	2,4,5	3,4,5	6,6,7	3,3,4
A_2	6,7,8	7,9,10	6,8,9	3,4,5	4,5,5	6,7,7	3,4,4
A_3	8,9,10	7,9,10	6,8,9	4,5,6	5,5,6	6,6,7	4,5,6
A_4	7,9,10	4,5,6	7,8,9	8,9,10	8,9,10	7,8,9	6,8,9
A_5	8,8,9	3,4,5	5,6,7	6,7,8	7,8,8	7,7,8	6,7,8
A_6	8,8,9	5,6,8	7,8,8	6,7,8	7,7,8	5,6,7	6,7,8
A_7	5,6,8	6,7,7	7,8,8	7,7,8	7,8,9	5,5,6	6,7,8
A_8	8,8,10	4,5,5	7,8,9	5,6,7	4,5,5	3,4,5	8,8,9
A_9	7,8,9	8,9,10	4,5,6	5,6,7	4,5,6	4,5,6	7,8,9
A_{10}	3,4,5	7,8,8	8,9,9	4,5,6	6,7,8	8,9,10	4,4,5
A_{11}	3,4,5	7,8,8	8,9,9	6,7,8	6,7,8	7,7,8	4,5,6

Each block is a triangular fuzzy number.

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MULTIPLE CRITERIA CHOICE OF R&D ORGANIZATION WITH THE AID OF STRUCTURAL METHODS

Abstract

Technology-intensive firms need to organize their R&D on a global scale. This is an important and complicated task that requires an explicit model and a thorough evaluation. Due to the complexity of the decisions about the global R&D organization and interrelations among the underlying issues, a structural approach is recommended. A detailed study of global R&D projects presented in the literature is used for structuring the problem with the aid of cognitive mapping. Based on this, two qualitative approaches: the Roberts model and the WINGS method are applied to and the most suitable solution.

Keywords: cognitive maps, Multiple Criteria Decision Aiding, research & development, structural methods, WINGS.

1. Introduction

R&D is a costly and risky activity but every company has to keep pace with the rapid development of technology. Running a project for a new product or a new technology/process demands extensive R&D and cooperation with other business units (e.g. marketing, manufacturing).

The organization of R&D should fit the tasks undertaken by the firm and be flexible fit should change when the environment changes. It is observed that an increasing amount of technical work is carried out abroad (Tidd and Bessant, 2009). Firms recognized that skills and talents needed to produce new technologies often develop locally and alocal presence facilitates and accelerates the

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processes of learning and knowledge absorption from foreign sources (De Meyer, 1993). This means that a large technology-intensive firm has to consider the organization of its R&D on a global scale.

All that makes the R&D organization one of the most important issues in the innovation process. The key question is how to support the decision of the choice of a suitable R&D organization. There is a broad spectrum of available Multiple Criteria Decision Aiding (MCDA) methods. Taking into account the complexity of the problem, the interrelations between its components and the interests of many stakeholders with various views and conflicting objectives, the author advocates the use of a structural (or systemic) approach. In this paper a series of related methods are presented. The use of a cognitive map is suggested to reveal the main objectives and their relations with the underlying issues. This technique provides a framework for quantitative approaches capable of differentiating among the potential solutions. The content of the decision model has been derived from a detailed study of global R&D projects (Chiesa, 2000).

The next section presents the different R&D structures observed in multinational companies. An example of problem setting is described in Section 3. The cognitive map technique is used to structure the problem and as a basis for a quantitative model (Section 4). It is followed by a more advanced approach using the WINGS method (Section 5). The last section contains conclusions.

2. Major Categories of Global R&D Structures

Chiesa examined 12 multinational companies covering different industries: automotive, chemicals, electro-mechanical, electronics, pharmaceutical, telecommunications, white goods, and concluded that multinational firms create global R&D organizations (Chiesa, 2000). All these firms conduct their R&D on an international scale and technology is important for their business. His paper focused on the management and organization of global R&D projects and aimed to classify the structures of global R&D. Chiesa discerned two major categories of global R&D structures: *specialization-based structure* and *integration-based structure*. In both, two sub-cases were found. Specialization-based structure can be divided into *center of excellence* and *supported specialization structure*, while integration-based structure, into *network structure* and *specialized contributors structure*. Below we briefly present these four solutions observed by Chiesa.

2.1. Specialization-based structure

This approach is based on the specialization of units and usually leads to concentration of the resources and R&D activities in one location. During the project

development process there is no cross-border management. Only one foreign lab (the firm's center of excellence) is assigned the full responsibility for developing a new product/process/technology on the basis of a global mandate.

2.1.1. Center of excellence

In this structure there is only one center in the firm that does R&D in a certain field and acts as the firm's center of excellence or center of competence in that field. The objective of this solution is to increase the R&D efficiency at the global level. The concentration of the resources allows to achieve economies of scale and greatly facilitates coordination. This structure is preferable when:

- The country hosting the center is a leading producer of market/technical knowledge useful for innovation in the given product/process/technology area.
- The product is global and markets are undifferentiated.
- The R&D resources of the firm in that field are concentrated or can be concentrated in one location.

Obviously, there are also disadvantages of such a structure. For example, the R&D center is isolated from divisions which are spread out around the world. Differences of culture and motivations between R&D and manufacturing and marketing units can create barriers when an innovation is introduced in the market.

Chiesa mentioned a few practical examples of such solutions. These are:

- A photoresistant and separation materials center of Hoechst in the U.S. (a case of undifferentiated products).
- A U.S. subsidiary of Alcatel responsible for the telecommunication transmission systems (the U.S. market is the most advanced in that field).
- Matsushita's microwave oven business unit whose R&D is concentrated in Japan (home country): this product is global and requires only small adaptation to the local market.

2.1.2. Supported specialization structure

This is a structure comprising a center of excellence that is assigned the global responsibility for R&D in a certain area, and a number of small units supporting the center. The small units are dispersed worldwide to supply market and technical information to the global center. Usually there are two kinds of auxiliary units:

- Units supporting product development; they are located close to the major customers to monitor trends and evolution, especially aesthetic and industrial design requirements of foreign markets.
- Units supporting research; they are located close to the technology centers of excellence in selected countries to monitor the technical progress and basic research advancements.

This kind of organization benefits from specialization and concentration without missing innovation opportunities that may arise worldwide. This structure is preferable when:

- Sources of innovation (customers, suppliers, research institutions, etc.) are dispersed but the degree of market differentiation is rather limited.
- The resources of the firm are concentrated or can be concentrated in one location. The advantages and disadvantages of this solution are similar to those previously mentioned, but they are weaker than in the previous case.

An example of supported specialization is that of Japanese consumer electronics companies (Toshiba, Matsushita). They have kept their R&D in their home country, while locating a number of small units in foreign countries close to key customers or technology centers of excellence.

2.2. Integration-based structure

This solution is based on integration of work of different units that are involved in all phases of the project, including project development. Global innovations are the result of joint work of these units. This kind of organization requires the management of dispersed resources and activities and a much stronger coordination.

2.2.1. Network structure

In this structure various foreign labs develop innovations in the same technological field or product area. The labs may undertake their own R&D initiatives and allocate a certain amount of resources to local projects. They are supervised centrally to avoid duplications and to coordinate the distributed activities. The coordinating unit is also responsible for leading joint R&D programs aimed to exploit the results across different markets. This structure is preferable when:

- The firm's resources are distributed and a permanent concentration in one location would result in suppressing pockets of technological excellence within the organization.
- Markets are differentiated and/or external sources of critical knowledge (key customers, centers of technological excellence) are dispersed.

The network structure can accelerate the process of learning because different units approach the same problem in different ways. It also promotes creativity as a result of internal competition among units.

The main problem with the network structure is the creation of mechanisms to coordinate these distributed activities and avoid duplications. Communication becomes a critical issue and frequent interactions between units are needed. It is obvious that this structure results in higher costs.

Among the firms that apply the network structure for their R&D are Ericsson, ABB and IBM.

2.2.2. Specialized contributors structure

This structure is based on structural division of labor among units. Each unit specializes in a certain technological discipline or product component. The structure is star-shaped with a central unit responsible for coordination and control of other units working on a specific part of the R&D program. An innovation appears as the result of the common effort of the component units. This structure is preferable when:

- A new product or production process can be divided into modules or subsystems.
- Different units can specialize in different technological or sub-product areas.

With this solution the specialized centers can be located close to external sources of knowledge and innovation. But again, as in the previous structure, problems with communication and coordination can occur. This structure puts also more demand on employee mobility.

A good example of this structure is that of the Ford Mondeo project ("a world car"). This R&D structure can be also convenient in the development of telecommunication systems.

3. Problem Setting

To show the potential of decision aiding in the organization of R&D we present the following case. A Europe-based company has a large R&D unit in Europe. Its manufacturing units are located in Europe and Asia. Most of the suppliers are located in Asia and some in Europe. Products are sold around the world, but the most important market is North America, with Europe the second, and Asia the third. The company is going to develop its R&D significantly and therefore it plans to design several global products aimed at moderately differentiated markets. The technological clusters that can be the sources of new knowledge are located in all developed regions, of which the most important is the Unites States, then Europe and Asia.

The company considers seven alternatives for the reorganization of its R&D. The first three options represent the 'center of excellence structure', with a center located: in Europe – alternative W1, in the U.S. – W2, or in Asia – W3. By "location in Europe" we mean that the existing center is to be expanded and modernized. The other locations need to be built from scratch. T' coordinated by the center: in Europe – alternative W4, in the U.S. – W5, or in Asia – W6. The last alternative consists in a transition to the network structure compounded from the independent labs in Europe, the U.S. and Asia. The European head office will be responsible for coordination.

4. Decision Support with Cognitive Map

4.1. Structuring the problem

The cognitive map has been elaborated according to the results of a study of global R&D projects (Chiesa, 2000). Its main role is to serve as a convenient tool for structuring the problem of choosing a suitable R&D organization. The map has been drawn so as to aid in decision making (Eden, 2004). The nodes at the bottom of the map represent the alternatives considered nodes in the middle correspond to the direct and indirect consequences of the chosen alternatives (direct consequences may be regarded as attributes of the alternatives). The top nodes reflect the ultimate goals, and therefore they have no outcoming arrows. In our example two conflicting goals are considered: 'lower R&D costs' and 'expanded and more effcient R&D'.

The cognitive map is a point of departure for the decision model proposed by Roberts (1976). That model uses quantitative information about the strength of influence between the concepts (nodes) on the map. If we wanted to better reflect user preferences we would need to take into account the different weights of concepts that appear on the map. This will lead us to the WINGS method.

Figure 1 presents the cognitive map for our R&D organization¹. To keep the figure clear, only one alternative (W1) and its influences are shown. The data gathered for all alternatives and presented in Table 1 allow the reader to easily reconstruct the full map. All valuations of influences presented on the map came from the same integer scale: from 1 – the lowest influence to 9 – the strongest influence. Obviously, if there is no direct influence (represented by 0), the corresponding arrow is not drawn.

Table 1 Impacts of the alternatives

Alternative	W1	W2	W3	W4	W5	W6	W7
Adaptation to manufacturing units localization	8	0	6	6	0	5	4
Adaptation to competence localization	9	2	2	7	3	4	4
Lower costs of duplications	9	9	9	8	8	8	0
Suppliers	5	2	7	6	3	8	8
Technology sources	3	7	5	5	9	7	7
Customers	6	8	4	7	9	5	7
Task duplications	0	0	0	2	2	2	9
Ease of coordination	8	8	8	7	7	7	2

Map has been drawn with the aid of CmapTools v. 5.05, Institute for Human and Machine Cognition (USA), http://cmap.ihmc.us.

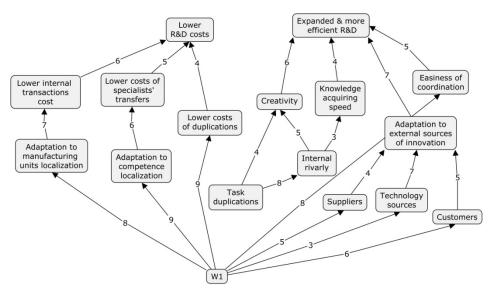


Figure 1. Cognitive Map of the R&D organization problem (Scale: integers from 1 – lowest influence to 9 – highest influence, 0 – no direct influence = no arrow)

4.2. Results of Cognitive Map analysis

Following Roberts' proposal one can calculate the total influence of each alternative on the ultimate goals. First, the partial influence for each path leading from the alternative to the ultimate goal is calculated by multiplying all the values along that path. The total influence of the given alternative on the chosen goal is the sum of the partial influences from all paths linking the alternative with the goal. A convenient way for doing calculations is to use the adjacency matrix corresponding to the map (see e.g.: Bang-Jensen and Gutin, 2008; Wallis, 2010). The results are presented in Table 2.

Table 2
Total impact of alternatives on final concepts (objectives)

Alternative	W1	W2	W3	W4	W5	W6	W7
Lower R&D costs	646	60	312	462	90	330	288
Expanded and more efficient R&D	542	719	621	1413	1595	1497	4062

The figures in Table 2 clearly show that alternative W2 is dominated by W4-W7, W3 is dominated by W4 and W6, and W5 is dominated by W7. Hence, the non-dominated set contains four alternatives: W1, W4, W6 and W7. Among them alternative W1 has the highest score for the goal 'Lower R&D costs' and W7 has the highest score for 'Expanded and more effcient R&D'.

The main drawback of the method is visible at first sight. Longer paths get higher scores which is counterintuitive. Consequently, differences in path lengths can bias the results. Another important weakness of this model is that it does not provide any way for differentiating the importance of the intermediate and ultimate criteria which are at the top of the map.

5. Decision Support with WINGS

WINGS (Weighted Influence Non-linear Gauge System) has been introduced to help solving complicated problems comprising many interrelated factors (Michnik, 2013a). It can be applied to multiple criteria problems as a special case. WINGS does not need the limiting assumption of criteria independence which is often unrealistic in practical applications.

5.1. WINGS Procedure

Based on the original paper (Michnik, 2013a) we briefly present here the steps of the WINGS procedure.

At the beginning the user selects n components that constitute the system and analyzes the important interdependencies among them. The result of this step is presented as a digraph in which nodes represent components and arrows represent influences between two nodes. In our case the cognitive map developed earlier will serve as a model of the system also for WINGS.

Then, the user chooses verbal scales for both: strength of components and their influence. Again, to keep consistency with the previous model, we will use integers from 1 to 9. Zero will represent no influence. In the case of multiple criteria decision making the strengths play the role of weights assigned to criteria at various levels. As the alternatives do not have weights assigned in advance, their strengths (weights) are all set to zero.

The next stages of the WINGS procedure are as follows:

- 1. All numbers are inserted into the direct strength-influence matrix **D**, which is an $n \times n$ matrix with elements d_{ij} .
 - Values representing strengths of components are inserted into the main diagonal, i.e. d_{ii} = strength of component i.
 - Values representing influences are inserted so that for $i \neq j$, $d_{ij} =$ influence of component i on component j.
- 2. The matrix \mathbf{D} is scaled according to the following formula:

$$\mathbf{C} = \frac{1}{s}\mathbf{D}$$

where the scaling factor is defined as the sum of all elements of the matrix **D**, i.e.:

$$s = \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij}$$

3. Calculate the total *strength-influence* matrix **T** from the formula (thanks to the scaling defined in Eq. (1) the series in the following formula converges, and thus the matrix **T** always exists):

$$T = C + C^2 + C^3 + ... = C (I - C)^{-1}$$

4. For each element in the system the row sum r_i and the column sum c_j of the matrix **T** are calculated:

$$r_i = \sum_{j=1}^n t_{ij}, \ c_j = \sum_{j=1}^n t_{ij},$$

where t_{ij} are the elements of the matrix **T**.

- 5. For each element in the system $r_i + c_i$ and $r_i c_i$ are calculated.
- 6. The WINGS output for each component is:
 - r_i total impact,
 - c_i total receptivity,
 - $r_i + c_i total involvement$,
 - sgn(r_i − c_i) indicates the role (position) of the component in the system: positive → influencing (cause) group, negative → influenced (result) group;
 - $|r_i c_i|$ indicates the level of the role.

Table 3

Weights for higher level intermediate and final concepts

Lower B&R costs	4
Lower internal transactions cost	4
Lower costs of specialists' transfers	4
Lower costs of duplications	2
Expanded and more efficient R&D	7
Adaptation to external sources of innovation	7
Knowledge acquisition speed	7
Creativity	7
Ease of coordination	4

5.2. Results of WINGS

To apply the WINGS method to our case we need to asses the weights of the criteria. Table 3 shows the numerical values of weights for the two ultimate criteria and for seven sub-criteria at the second-highest level.

Now we use all the data: influences assigned in the previous model (Figure 1 and Table 1) and weights from Table 3 to do calculations in WINGS. In Table 4 there are values of 'total involvement' (r + c) for all alternatives (they are equal to 'total impact' (r) because the alternatives have no incoming arrows). Ordering the alternatives from the highest value of 'total involvement' to the lowest gives a ranking of

the alternatives. When we compare it to the results of the previous model we can observe that WINGS ranks the alternatives differently. Alternative W1 is the leader, followed by W4, W6 and W7 respectively. However, there are some similarities. In both models W1 and W4 rank high while W2 and W3 rank lowest.

Table 4
Ranking of alternatives

Alternative	W1	W2	W3	W4	W5	W6	W7
r + c	0,1299	0,0819	0,0933	0,1093	0,0934	0,1048	0,0937
Place in ranking	1	7	6	2	5	3	4

It is possible to acquire some additional knowledge from the WINGS output. For example, it might be useful to analyze the role of the elements that constitute the system of R&D organization. The influences of alternatives would distort the evaluation so they need to be excluded from the network.

WINGS calculations have been performed for 17 elements included in the model of R&D reorganization. The final scores have been re-scaled so that the highest absolute value of r + c or r - c is set to 10. The complete results are presented in Table 5 and in Figure 2 (for clarity, only selected points are labeled on the map). It is obvious that the ultimate goals have the highest values of both 'involvement' (r + c) and 'role' (r - c). Among other elements, 'Adaptation to external sources of innovation' and 'Creativity' have the highest involvement which suggests that these issues are very important for a successful R&D reorganization and deserve the highest attention.

Table 5
The involvement and role of the elements in WINGS model

No.	Element	r+c	r-c
1	Adaptation to manufacturing units localization	1,858	2,920
2	Adaptation to competence localization	1,581	2,485
3	Lower costs of duplications	1,000	0,000
4	Suppliers	1,096	1,722
5	Technology sources	1,918	3,014
6	Customers	1,370	2,153
7	Adaptation to external sources of innovation	9,712	-3,518
8	Task duplications	3,187	5,008
9	Creativity	7,696	-1,212
10	Internal rivarly	4,134	0,303
11	Knowledge acquiring speed	5,559	0,425
12	Easiness of coordination	3,369	2,104
13	Lower internal transactions cost	5,377	-0,326
14	Lower costs of specialists' transfers	4,862	-0,338
15	Lower costs of duplications	2,550	1,632
16	Lower R&D costs	6,086	-6,374
17	Expanded and more efficient R&D	10,000	-10,000

'Task duplications' has the highest value of r-c. It is followed by 'Technology sources' and 'Adaptation to manufacturing units localization'. This means that these elements are among the most sensitive points of the system. It should be noted that the element 'Task duplications' is related to higher costs but its positive effects overcome that drawback.

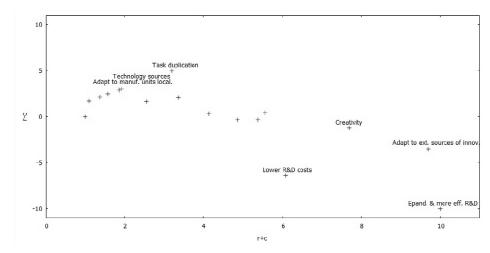


Figure 2. The map presenting the involvement and role of elements in the WINGS model (only selected points are labeled)

6. Conclusions

Reorganization of R&D, as any other significant change in a firm, is a complicated organizational process that comprises many important issues and involves many people. Previous studies have shown how various aspects need to be considered when decisions about the most suitable way of conducting R&D are to be taken. Since there are strong relationships between the elements that are important for the underlying decisions, it is unlikely that the classic multiple criteria approach can be a suitable model for decision-aiding. Instead, the structural (or systemic) approach can be much more adequate, especially when there are many subjects with various views and different interests participating in a process.

In this paper a decision-aiding procedure for the choice of R&D organization has been proposed. It starts with the construction of a cognitive map which is a powerful technique for structuring the problem and coordinating the various views of the participants. Since a cognitive map by itself is a qualitative tool with limited ability to differentiate among the decision alternatives (see e.g. Michnik, 2013b) it has been proposed to complement it by a quantitative assessment of influences that formally led to the model proposed by Roberts. However, this approach has also some drawbacks, so the WINGS method is ad-

vocated as the most suitable method to help in decision-making. It also gives more insight into the problem by showing explicitly those issues that are the strongest influencers and those that are the most susceptible. It is worth noting that WINGS is technically simpler than other similar methods, such as ANP.

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INTERACTIVE APPROACH APPLICATION TO STOCHASTIC MULTIOBJECTIVE ALLOCATION PROBLEM – A TWO-PHASE APPROACH

Abstract

In this paper a stochastic multiobjective allocation problem is considered. We assume that a particular resource should be allocated to T projects. Depending on the amount of allocated resource it is possible (with known probabilities) to obtain a specified level of each goal. The considered criteria are divided into two groups. The first group consists of financial criteria, the second one, of qualitative criteria, representing the degree to which the projects contribute to reaching strategic goals. We propose a two-phase procedure for identifying the strategy that should be implemented by a decision maker. Our technique combines multiobjective dynamic programming and interactive approach. First, efficient strategies are identified using Bellman's principle of optimality adapted to the multiobjective problem. Next, a dialog procedure is applied to identify the solution that satisfies the decision maker. A numerical example is presented to show the applicability of the procedure.

Keywords: stochastic allocation problem, multiobjective dynamic programming, interactive approach, stochastic dominance.

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1. Introduction

Various types of stochastic allocation problems are considered in the literature. For example, sensor allocation management as a stochastic dynamic programming resource allocation problem is considered in Johansson et al. (2005). Techniques for approximately solving a certain class of resource allocation problems that arise in the context of stochastic sequential decision making problems that are computationally efficient are considered in Hariharan (2009). Another problem can be formulated as follows: A given quantity of a resource is to be allocated to several activities. The amount of the resource allocated to each activity is used to satisfy randomly occurring stochastic demands. The system operates as long as all the demands can be met (Mendelson, Pliskin, Yechiali, 1980). An application of dynamic resource allocation in wireless communication using stochastic optimization is described in Li (2012).

The single objective deterministic allocation problem, formulated as above, has been described and solved in Bellman (1957) and Bellman, Dreyfus (1962). A dynamic programming approach has been applied. A bi-objective allocation problem was analyzed in Trzaskalik (2008). The way in which the vector version of Bellman's optimality principle can be used for identifying the whole set of non-dominated solutions has been shown.

A stochastic multiobjective allocation problem (considered in our previous paper Nowak, Trzaskalik, 2014) can be described as follows. A company has a limited amount of a particular resource that can be allocated to various projects. We assume that the decision maker has defined *K* goals that he/she would like to achieve. For each goal several levels of achievement have been specified. The problem is a stochastic one: if a given amount of the resource is allocated to a given project, a given level of goal is achieved with a given probability. Hence, we are to decide which projects should be implemented and what should be the intensity of their realization.

The allocation problem considered in the present paper differs from the previous one. We assume that all projects are similar and provide an identical financial return. However, the return depends on the way in which the resource is split among projects. Let $(x_1, x_2, ..., x_T)$ mean that x_1 units of the resource are allocated to project 1, x_2 units are allocated to project 2 and so on. According to our assumption, the financial return of the allocation (x, 0, ..., 0) is the same as for (0, x, ..., 0) and (0, 0, ..., x). Since the financial result, while important, does not always reflect strategic importance, we also take into account non-financial criteria. We assume that the degree to which the projects contribute in reaching strategic goals is not the same, even when the financial return is identical.

Our goal is to propose a two-phase procedure for identifying the best allocation of the resource. Our method combines multiobjective dynamic programming and interactive approach. Since for most companies, financial result is of crucial importance, we propose to use a two-phase procedure. First, allocations are evaluated with respect to financial criteria using Monte-Carlo simulation. Then, non-dominated allocations with respect to financial criteria are identified using Bellman's principle of optimality adapted to the multiobjective problem. In the second phase experts are asked to evaluate non-dominated allocations with respect to non-financial criteria.

The evaluations of allocations with respect to financial criteria obtained by Monte-Carlo simulations are random variables. On the other hand, the evaluations with respect to non-financial criteria are measured on an ordinal scale and are deterministic. As a result, we have here a multicriteria mixed stochastic-deterministic problem. In our paper we propose an interactive procedure for identifying the final solution of the problem. A candidate solution is presented to the decision maker in each iteration. If he/she is satisfied with the proposal, the procedure ends, otherwise he/she is asked to express his/her preferences specifying the values that the criteria should achieve, or at least indicating the criterion that should be improved.

The rest of the paper is organized as follows. In section 2 dynamic programming with a partially ordered criteria space is considered. A stochastic allocation problem as an example of dynamic programming with partially ordered criteria space is presented in section 3. An interactive procedure for identifying the final solution is described in section 4. In section 5 a numerical example is presented. The last section consists of final remarks.

2. Dynamic programming with partially ordered criteria space

The presented description of the discrete dynamic decision process comes from Trzaskalik (1990; 1998). We consider multiobjective, multiperiod decision process, which consists of *T* periods.

We assume that for $t \in \overline{1,T}$:

 \mathbf{Y}_t is the set of all feasible states at the beginning of the period t,

 \mathbf{Y}_{T+1} is the set of all feasible states at the end of the process,

 $\mathbf{X}_t(y_t)$ is the set of all feasible decisions for the period t and the state y_t ,

 $\mathbf{D}_t(y_t)$ is the set of all period realizations in the period t, defined as follows:

$$\mathbf{D}_{t}(y_{t}) = \left\{ d_{t}(y_{t}, x_{t}) \colon y_{t} \in \mathbf{Y}_{t}, x_{t} \in \mathbf{X}_{t}(y_{t}) \right\}$$
(1)

 $\Omega_t: D_t \to \mathbf{Y}_{t+1}$ is a given transformation.

D is a set of all process realizations, defined as follows:

$$\mathbf{D} = \left\{ d = \left(d_1, \dots, d_T \right) \colon \forall_{t \in \overline{1,T}} \ y_{t+1} = \Omega_t \left(y_t, x_t \right) \right\}$$
 (2)

The sets $\mathbf{Y}_1, ..., \mathbf{Y}_{T+1}, \mathbf{X}_1(y_1), ..., \mathbf{X}_T(y_T)$ and functions $\Omega_1, ..., \Omega_T$ are identified. We assume that a partially ordered criteria space $(\mathbf{W}, \leq, \circ)$ is given, which consists of a set \mathbf{W} , the preference relation \leq (named "not worse than") and the binary operator \circ .

Let $V \subset W$. We denote by max V the set of all maximal elements of V, defined as follows:

$$\max \mathbf{V} \{ \hat{\mathbf{v}} \in \mathbf{V} : \sim \exists_{\overline{\mathbf{v}} \in \mathbf{V}} \ \hat{\mathbf{v}} < \overline{\mathbf{v}} \}$$
 (3)

If $\hat{v} < \overline{v}$, we say, that \hat{v} is worse than \overline{v} .

We consider period criteria functions $f_t : \mathbf{D}_t \to \mathbf{W}$. The multiperiod criteria function F is defined as follows:

$$F = f_1 \circ \left(f_2 \circ \left(\dots \left(f_{T-1} \circ f_T \right) \right) \right) \tag{4}$$

A realization $\hat{d} \in \mathbf{D}$ is said to be efficient, iff:

$$\sim \exists_{\overline{d} \in \mathbf{D}} F(\hat{d}) < F(\overline{d}) \tag{5}$$

The set of all efficient realizations is denoted by $\hat{\mathbf{D}}$.

The dynamic programming problem with a partially ordered criteria space is formulated as follows (Trzaskalik, Sitarz, 2002; Trzaskalik, Sitarz, 2007): find $\hat{\mathbf{D}}$ in the decision space and $F(\hat{\mathbf{D}})$ in the criteria space.

The optimality equations can be written as follows (Bellman, 1957; Trzaskalik, Sitarz, 2002):

$$G_T^*(y_T) = \max\{f_T(y_T, x_T) : x_T \in \mathbf{X}_T(y_T)\}$$
 (6)

$$G_{\cdot}^{*}(y_{\cdot}) = \max \left\{ F_{\tau}(y_{\cdot}, x_{\cdot}) \circ G_{\cdot+1}^{*}(y_{\cdot}, x_{\cdot}) : x_{\tau} \in X_{\tau}(y_{\tau}) \right\}$$
(7)

for t = T - 1, ..., 1.

Theorem 1 (Trzaskalik, Sitarz, 2002). The condition:

$$F(\hat{\mathbf{D}}) = \max \bigcup_{y_1 \in \mathbf{Y}_1} G_1^*(y_1)$$
 (8)

holds. By applying Theorem 1 we can find the sets $F(\hat{\mathbf{D}})$ and $\hat{\mathbf{D}}$.

3. Stochastic allocation problem as an example of dynamic programming with partially ordered criteria space

Let us consider the set of probability sequences, defined in the following way:

$$\mathbf{W} = \left\{ (p_0, p_1, \dots, p_n) : n \in \mathbf{N}, p_n > 0, p_i \ge 0, \sum_{i=0}^n p_i = 1 \right\}$$
 (9)

where $p_i = P(X = i)$, $n = \max\{k \in \mathbb{N}, p_k > 0\}$. For simplicity we assume that random variables take only nonnegative integer values. This is an example of a partially ordered criteria space.

Let $p, q \in \mathbf{W}$, $p = (p_1, ..., p_n)$, $q = (q_1, ..., q_m)$. We define the operator \circ as follows:

$$p \circ q = (r_0, r_1, \dots, r_{n+m})$$
 (10)

where
$$r_i = \sum_{\substack{k,l \ k+l=i}} p_k q_l$$
 for $i = \overline{1, n+m}$.

To define the relation \leq we use the FSD (First Stochastic Dominance) and SSD (Second Stochastic Dominance) relations:

$$p \le q \iff q \text{ FSD } p \lor q \text{ SSD } p$$
 (11)

where:

$$q \text{ FSD } p \iff \forall_{i \in \overline{0,n}} \sum_{k=0}^{i} q_k \le \sum_{k=0}^{i} p_k$$
 (12)

$$q \text{ SSD } p \quad \Leftrightarrow \quad \forall_{i \in \overline{0,n}} \sum_{k=0}^{i} \sum_{l=0}^{k} q_{l} \le \sum_{k=0}^{i} \sum_{l=0}^{k} p_{l}$$
 (13)

The stochastic allocation problem, considered in this paper can be regarded as a T – stage decision problem in a partially ordered criteria space. The number of stages is determined by the number of analyzed projects. In the stage t ($t \in \overline{1,T}$) the decision on the number of units allocated to a particular project is made.

The set \mathbf{Y}_t of feasible states y_t in the consecutive stages $t \in \overline{1,T}$ is defined as follows:

$$\mathbf{Y}_t = \left\{ y_t \in \mathbf{N}^0 : 0 \le y_t \le n_0 \right\} \tag{14}$$

The set of feasible decisions for the consecutive states $y_t \in \mathbf{Y}_t$ for $t \in \overline{1,T}$ is defined in the following way:

$$\mathbf{X}_{t}(y_{t}) = \left\{ x_{t} \in \mathbf{N}^{0} : 0 \le x_{t} \le y_{t} \right\}$$
 (15)

For $t \in \overline{1,T}$ the transition functions are defined as follows:

$$y_{t+1} = y_t - x_t (16)$$

Each of the *K* criteria functions is an element of the set **W**, described by formula (1). The criteria space \mathbf{W}^K is the product of *K* structures **W**. The operator \circ^K and relation \leq^K are defined as follows:

$$(p^{1}, p^{2}, ..., p^{K}) \circ^{K} (q^{1}, q^{2}, ..., q^{K}) \stackrel{\text{def}}{=} (p^{1} \circ q^{1}, p^{2} \circ q^{2}, ..., p^{K} \circ q^{K})$$

$$(p^{1}, p^{2}, ..., p^{K}) \leq^{K} (q^{1}, q^{2}, ..., q^{K}) \stackrel{\text{def}}{=} p^{1} \leq q^{1} \wedge p^{2} \leq q^{2} \wedge ... \wedge p^{K} \leq q^{K}$$

$$(18)$$

The relation \leq^K holds, if the FSD or SSD relations hold for each criterion (Trzaskalik, Sitarz, 2004).

4. Interactive procedure

Solving a multicriteria problem is possible when the information about the preferences of the decision maker is available. Interactive approach is often used for this purpose. The procedure presented here is a modification of the INSDECM method (Nowak, 2006; Nowak, Trzaskalik, 2013).

The general scheme of each interactive procedure is similar. In each iteration a single solution or a subset of feasible solutions is presented to the decision maker, who evaluates the proposals and specifies the way in which the solution should be improved. As in our problem the evaluation of each alternative with respect to each criterion is represented by a probability distribution, we must choose measures that will be used during the dialog phase of the procedure. These can be either measures of central tendency, or measures of dispersion. At least one measure must be used for each criterion. The measures should be chosen by the decision maker in the initial phase of the procedure.

The final solution is identified in a stepwise manner. In each iteration the potency matrix is generated and presented to the decision maker. It consists of two rows: the first grouping the worst (pessimistic), and the second – the best (optimistic) values of the considered measures attainable independently within the set of alternatives.

In addition to the potency matrix, a candidate solution is presented to the decision maker. As the criteria differ in nature, we use lexicographic approach to identify it. At the initial phase of the procedure the decision maker is asked to define a hierarchy of criteria. This information is used in each iteration to select the candidate solution.

Let $\widehat{\mathbf{D}}$ be the set of efficient solutions, $\mathbf{D}^{(l)}$ – the set of solutions analyzed in iteration l, M – the number of measures that are analyzed, and $\mathbf{G}^{(l)}$ – the potency matrix:

$$\mathbf{G}^{(l)} = \begin{bmatrix} \underline{g}_{1}^{(l)} & \cdots & \underline{g}_{m}^{(l)} & \cdots & \underline{g}_{M}^{(l)} \\ -(l) & -(l) & -(l) & -(l) \\ \underline{g}_{1} & \cdots & \underline{g}_{m} & \cdots & \underline{g}_{M} \end{bmatrix}$$
(19)

where: $\underline{g}_{m}^{(l)}$ is the worst value of the *m*-th measure attainable in the set of solutions analyzed in iteration *l*, and $\underline{g}_{m}^{(l)}$ is the best one.

The procedure for identifying the final solution operates as follows: *Initial phase:*

- 1. Ask the decision maker to define the hierarchy of criteria.
- 2. Ask the decision maker to specify measures that will be used to evaluate solutions with respect to the financial criteria.
- 3. l = 1. $\mathbf{D}^{(1)} = \widehat{\mathbf{D}}$.

Iteration l

- 1. Identify potency matrix $\mathbf{G}^{(l)}$.
- 2. Identify the candidate solution taking into account the hierarchy of criteria defined by the decision maker in the initial phase.
- Present the potency matrix and the candidate solution to the decision maker.
 Ask the decision maker if he/she is satisfied with the candidate solution. If the answer is "yes", go to (8).
- 4. Ask the decision maker if he/she would like to define the aspiration levels for criteria. If the answer is "no" go to (6).
- 5. Ask the decision maker to specify aspiration levels $\widetilde{g}_m^{(l)}$ for m = 1, ..., M. Identify the set $\mathbf{D}^{(l+1)}$ including the solutions satisfying the decision maker's requirements. If $\mathbf{D}^{(l+1)} = \emptyset$ report it to the decision maker and go to (4), otherwise go to (7).
- 6. Ask the decision maker to indicate the index m of the measure for which the pessimistic value is unsatisfactory. Identify the set $\mathbf{D}^{(l+1)}$ grouping the solutions for which the value of the m-th measure is better than the current pessimistic value $\underline{\mathbf{g}}_{m}^{(l)}$.
- 7. Set l = l + 1 and go to (2).
- 8. Ask the decision maker to indicate the index m of the measure that should achieve the optimistic value, select the solution for which the m-th measure is equal to the optimistic value $g_m^{-(l)}$.

5. Numerical example

The company is going to allocate 6 units of a resource among 3 projects. Three financial criteria are considered:

- Criterion 1: Net Present Value generated by the project.
- Criterion 2: Revenues made on new market.
- Criterion 3: Revenues made from a new product.

Additionally, the projects are evaluated with respect to the following qualitative criteria, representing the degree to which the projects contribute to reaching strategic goals:

- Criterion 4: Strategic fit.
- Criterion 5: The use of core competencies.
- Criterion 6: Improving customer loyalty.

Our goal is to determine the amount of resource that should be allocated to each project. The first phase of the procedure starts with running simulation experiments. As we assume that the results generated by each project are identical, and the total value obtained for the project portfolio is the sum of values obtained for the individual projects, it is enough to run the simulations for a single project. The results of the simulations is presented in Table 1.

Table 1
The results of the simulation

Number of units	Criter	rion 1	Criter	rion 2	Crite	rion 3
allocated to project	Value (10 ⁵ EUR)	Probability	Value (10 ⁵ EUR)	Probability	Value (10 ⁵ EUR)	Probability
0	0.0	1.0	0.0	1.0	0.0	1.0
1	0.0	0.5	0.0	0.3	0.0	0.8
	1.0	0.5	0.1	0.7	0.1	0.2
2	0.0	0.4	0.0	0.3	0.0	0.7
	1.0	0.5	0.1	0.5	0.1	0.2
	2.0	0.1	0.2	0.2	0.2	0.1
3	0.0	0.3	0.0	0.2	0.0	0.6
	1.0	0.5	0.2	0.4	0.2	0.2
	2.0	0.2	0.2	0.4	0.2	0.2
4	0.0	0.2	0.0	0.1	0.0	0.5
	1.0	0.5	0.1	0.3	0.1	0.2
	2.0	0.2	0.2	0.4	0.2	0.2
	3.0	0.1	0.2	0.2	0.2	0.1
5	0.0	0.2	0.0	0.1	0.0	0.4
	1.0	0.4	0.1	0.2	0.1	0.2
	2.0	0.2	0.2	0.4	0.2	0.2
	3.0	0.2	0.3	0.3	0.3	0.2
6	0.0	0.1	0.0	0.0	0.0	0.3
	1.0	0.3	0.1	0.1	0.1	0.3
	2.0	0.3	0.2	0.4	0.2	0.2
	3.0	0.3	0.3	0.5	0.3	0.2

By applying the dynamic programming procedure we obtain the following non-dominated allocations:

All the computations are presented in the APPENDIX.

While the financial results for all allocations from the same profile are exactly the same, the evaluations with respect to qualitative criteria may be different. In the second phase of the procedure, experts are asked to rank the non-dominated allocations with respect to qualitative criteria. Each expert evaluates an allocation with respect to a single criterion. The results of this phase are presented in Table 2.

Table 2 Rankings of resource allocations based of experts' evaluations

Rank	Criterion 4	Criterion 5	Criterion 6
1	(6, 0, 0)	(0, 5, 1)	(0, 0, 6)
2	(5, 1, 0)	(1, 5, 0)	(0, 1, 5)
3	(1, 5, 0)	(0, 6, 0)	(1, 0, 5)
4	(0, 6, 0)	(5, 0, 1)	(5, 0, 1)
5	(0, 5, 1)	(1, 4, 1)	(5, 1, 0)
6	(4, 1, 1)	(5, 1, 0)	(6, 0, 0)
7	(5, 0, 1)	(6, 0, 0)	(0, 5, 1)
8	(1, 4, 1)	(4, 1, 1)	(0, 6, 0)
9	(0, 0, 6)	(1, 0, 5)	(1, 5, 0)
10	(0, 1, 5)	(0, 0, 6)	(1, 1, 4)
11	(1, 0, 5)	(1, 1, 4)	(1, 4, 1)
12	(1, 1, 4)	(0, 1, 5)	(4, 1, 1)

Finally, the interactive procedure proposed in the previous section is used to identify the final solution. The dialog with the decision maker is conducted as follows:

Initial phase:

The hierarchy of the criteria defined by the decision maker is as follows: Criterion 1, Criterion 3, Criterion 2, Criterion 6, Criterion 5, Criterion 4 The decision maker defines also the data that should be presented:

- Criterion 1: probability that NPV is not less than $2 \cdot 10^5$ EUR.
- Criterion 2: probability that revenues made on new market are not less than $0.2 \cdot 10^5$ EUR.
- Criterion 3: probability that revenues made from a new product are not less than $0.2 \cdot 10^5$ EUR.
- Project's ratings with respect to qualitative criteria.

Table 3

Iteration 1:

- 1. The initial solution is identified taking into account the hierarchy defined by the DM. The allocations (4, 1, 1), (1, 4, 1) and (1, 1, 4) are the best with respect to criterion 1. Since they are equally evaluated with respect to criteria 2 and 3, in order to determine the initial solution, we consider criterion 6, which is next in the hierarchy. Among these three the allocation (1, 1, 4) is rated the best according to this criterion, and as a result it is the initial solution.
- 2. The potency matrix and the actual solution are presented to the decision maker (Table 3).

Potency matrix no. 1

			-			
Solution	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion 6
Optimistic	0.725	0.922	0.440	1	1	1
Pessimistic	0.600	0.840	0.392	12	12	12
(1, 1, 4)	0.725	0.922	0.392	12	11	10

- 3. The DM is satisfied with the results for criteria 1-3, but not with the evaluations with respect to the qualitative criteria. He asks to take into account only these solutions which rank not less than 8 with respect to criterion 6.
- 4. The set of allocations satisfying the requirement formulated by the DM is identified:

(6, 0, 0); (0, 6, 0); (0, 0, 6); (5, 1, 0); (5, 0, 1); (1, 0, 5); (0, 5, 1); (0, 1, 5)

5. New potency matrix is constructed (Table 4). The DM is asked if he accepts the new results. The answer is YES.

Potency matrix no. 2

	Solution	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion 6
Op	otimistic	0.600	0.900	0.440	1	1	1
Pe	ssimistic	0.600	0.840	0.400	11	12	8

Iteration 2:

1. New proposal is identified taking into account the hierarchy of criteria: (0, 1, 5). The evaluations of the proposal are presented to the DM (Table 5).

Table 5

Table 4

Proposal solution in iteration 2

Solution	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion 6
(0, 1, 5)	0.600	0.840	0.440	10	12	2

- 2. The DM is satisfied with the results for criteria 1, 2, 3 and 6, but not with the evaluations with respect to criteria 4 and 5. He asks to take into account only those solutions which rank not less than 8 with respect to criterion 5.
- 3. The set of allocations satisfying the requirement formulated by the DM is identified:

$$(6, 0, 0); (0, 6, 0); (5, 1, 0); (5, 0, 1); (0, 5, 1)$$

4. New potency matrix is constructed (Table 6). The DM is asked if he accepts the new results. The answer is YES.

Table 6

Potency matrix no. 3

Solution	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion 6
Optimistic	0.600	0.900	0.440	1	1	4
Pessimistic	0.600	0.840	0.400	7	7	8

Iteration 3:

1. New proposal is identified taking into account the hierarchy of criteria: (5, 0, 1). The evaluations of the proposal are presented to the DM (Table 7).

Table 7

Proposal solution in iteration 3

Solution	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion 6
(0, 1, 5)	0.600	0.840	0.440	7	4	4

2. The DM is satisfied with the results and accepts the allocation (0, 1, 5) as a final solution.

6. Final remarks

The allocation problem considered in this paper can be used to describe a wide range of real-world problems. Dynamic programming is an efficient tool for solving it. However, in a multiobjective environment it must be used together with a procedure for the identification of the final solution. We propose to use an interactive method, in which preference information is obtained in a stepwise manner. This allows the decision maker to obtain more insight into trade-offs among different criteria. It is often pointed out that decision makers put much reliance in solutions generated via an interactive procedure, and as a result, such solutions have better chances of being implemented.

In our problem the decision maker defines his/her requirements in relation to the process realization. The nature of the problem is hierarchical. The advantage of the two-phase approach consists in reducing the number of alternatives evaluated by the experts. They evaluate only those which are likely to be accepted by the decision maker.

Appendix

					$\max \{ (a) [(0,5;0,5) \times (0,3;0,7) \times (0,8);0,2)]$ (b) $[(0,5;0,5) \times (0,3;0,7) \times (0,8);0,2)] \}$		II			
$G_3^*(y_3) = \max \{ F_3(y_3, x_3) : x_3 \in X_3(y_3) \}$	$x_3*(0) = 0$ $x_3*(1) = 1$ $x_3*(2) = 2$ $x_3*(3) = 3$ $x_3*(4) = 4$ $x_3*(5) = 5$ $x_3*(6) = 6$	$G_2^*(y_2) = \max \{ F_2(y_2, x_2) \bullet G_3^*(y_2 - x_2) : x_2 \in X_2(y_2) \}$) × (1)] }		$\max \{ (a) [(1) \times (1) \times (1)] \bullet [(0.5; 0.5) \times (0.3; 0.7) \times (0.8); 0.2)] = (b) [(0.5; 0.5) \times (0.3; 0.7) \times (0.8); 0.2)] \bullet [(1) \times (1) \times (1)] \}$	(1) = 1	$\max \{ (a) [(1) \times (1) \times (1)] \bullet [(0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1)]$ (b) $[(0.5; 0.5) \times (0.3; 0.7) \times (0.8); 0.2)] \bullet [(0.5; 0.5) \times (0.3; 0.7) \times (0.8); 0.2)]$ (c) $[(0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1)] \bullet [(1) \times (1) \times (1) \times (1)] \}$			$x_2^{*}(2) = 1$
$G_3^*(y_3) = \max$; 0.2)] } 2) ×(0,7; 0.2; 0,1)] } 3; 0,4; 0.2; 0,2)] } 3; 0,4; 0.2) ×(0,5; 0.2; 0.2; 0.2; 0.3; 0.3; 0.4; 0.2; 0.3; 0.3; 0.3; 0.3; 0.3; 0.3; 0.3; 0.3	$G_2^*(y_2) = \max \{ F_2 (y_2) \}$	\times (1)] \bullet [(1) \times (1) \times (1)]} = {[(1) \times (1)	$x_2^*(0) = 0$	$\max \{ (a) [(1) \times (1) \times (1)] $ (b) $[(0,5,0,5) \times (0,3)]$	(9,2)] $ x_2*(1) = 0 or x_2*(1) = 1 $	$\max \{ (a) [(1) \times (1) \times (1)] \bullet \\ (b) [(0,5,0,5) \times (0,3,0)] \bullet \\ (c) [(0,4,0,5,0,1) \times (0,0,1)] \bullet $; 0,2) × (0,7; 0,2; 0,1)] 0,42; 0,49) × (0,64; 0,32; 0,04)] ; 0,2) × (0,7; 0,2; 0,1)] }	ve have:) × (0,64; 0,32; 0,04)] }
	$\begin{array}{l} G_3^*(0) = \{F_3(0,0)\} = \{[(1)\times(1)\times(1)]\} \\ G_3^*(1) = \{F_3(1,1)\} = \{[(0,5;0.5)\times(0.3;0.7)\times(0.8);0.2)]\} \\ G_3^*(2) = \{F_3(2,2)\} = \{[(0,4;0.5;0.1)\times(0.3;0.5)\times(0.5;0.2)\times(0.7;0.2;0.1)]\} \\ G_3^*(3) = \{F_3(3,3)\} = \{[(0,2;0.5;0.1)\times(0.2;0.4;0.4)]\times(0.6;0.2;0.2)]\} \\ G_3^*(4) = \{F_3(4,4)\} = \{[(0,2;0.5;0.2)\times(0.1;0.3;0.4;0.2)\times(0.5;0.2)]\} \\ G_3^*(5) = \{F_3(5,5)\} = \{[(0,2;0.5;0.2)\times(0.1;0.2;0.3;0.4)\times(0.5;0.2;0.2)]\} \\ G_3^*(6) = \{F_3(6,6)\} = \{[(0,1;0.3;0.3)\times(0;0.1;0.5;0.5)\times(0.5;0.5)\}\} \end{array}$		$G_2*(0) = \max \{ F_2(0, 0) \bullet G_3*(0) \} = \max \{ [(1) \times (1) \times (1)] \bullet [(1) \times (1) \times (1)] \} = \{ [(1) \times (1) \times (1)] \}$	$G_2*(0) = \{[(1) \times (1) \times (1)]\}$	$G_2*(1) = \max \{ F_2(1, 0) \bullet G_3*(1) = F_2(0, 1) \bullet G_3*(0) \}$	As a = b, we have: $G_2*(1) = \{ \ [(0.5; 0.5) \times (0.3; 0.7) \times (0.8); 0.2)] \ \}$	$G_2*(2) = \max \{ F_2(2, 0) \bullet G_3*(2) = F_2(1, 1) \bullet G_3*(1) \}$ $F_2(0, 2) \bullet G_3*(0) \}$	= max { (a) [(0,4; 0,5; 0,1) × (0,3; 0,5; 0,2) × (0,7; 0,2; 0,1)] (b) [0,25; 0,5; 0,25) × (0,09; 0,42; 0,49) × (0,64; 0,32; 0,04)] (c) [(0,4; 0,5; 0,1) × (0,3; 0,5; 0,2) × (0,7; 0,2; 0,1)] }	As b (FSD, FSD, SSD) a and b (FSD, FSD, SSD) c, we have:	$G_2*(2) = \{ [0.25; 0.5; 0.25) \times (0.09; 0.42; 0.49) \times (0.64; 0.32; 0.04)] \}$

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x_2^*(4) = 0 \text{ or } x_2^*(4) = 1 \text{ or } x_2^*(4) = 3 \text{ or } x_2^*(4) = 4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            x_2^*(3) = 0 \text{ or } x_2^*(3) = 1 \text{ or } x_2^*(3) = 2 \text{ or } x_2^*(3) = 3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  \begin{array}{l} \max \left\{ (\mathbf{a}) \left[ (1) \times (1) \times (1) \right] \bullet \left[ (0.2; 0.4; 0.2; 0.2) \times (0.1; 0.2; 0.3; 0.4) \times (0.4; 0.2; 0.2; 0.2) \right] \\ (\mathbf{b}) \left[ (0.5; 0.5) \times (0.3; 0.7) \times (0.8); 0.2) \right] \bullet \left[ (0.2; 0.5; 0.1) \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.2) \right] \\ (\mathbf{c}) \left[ (0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1) \right] \bullet \left[ (0.3; 0.5; 0.2) \times (0.2; 0.4; 0.4) \right] \\ (\mathbf{d}) \left[ (0.3; 0.5; 0.2) \times (0.2; 0.4; 0.4) \right] \bullet \left[ (0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1) \right] \\ (\mathbf{d}) \left[ (0.2; 0.5; 0.2; 0.1) \times (0.1; 0.3; 0.4; 0.2) \times (0.3; 0.2; 0.2) \times (0.3; 0.3; 0.7) \times (0.8; 0.2) \right] \\ (\mathbf{d}) \left[ (0.2; 0.4; 0.2; 0.2) \times (0.1; 0.2; 0.3; 0.4) \times (0.4; 0.2; 0.2; 0.2) \right] \bullet \left[ (0.3; 0.3; 0.2) \times (0.1; 0.2; 0.3; 0.4) \times (0.4; 0.2; 0.2; 0.2) \right] \\ (\mathbf{d}) \left[ (0.2; 0.4; 0.2; 0.2) \times (0.1; 0.2; 0.3; 0.4) \times (0.4; 0.2; 0.2; 0.2) \right] \bullet \left[ (1) \times (1) \times (1) \right] \end{array} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  \max \{ (a) \ [(1) \times (1) \times (1)] \bullet [(0.2; 0.5; 0.2; 0.1) \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.2; 0.2; 0.1)] \\ (b) \ [(0.5; 0.5) \times (0.3; 0.7) \times (0.8); 0.2)] \bullet [0.3; 0.5 \times (0.2) \times (0.2; 0.4; 0.4)] \\ (c) \ [(0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1)] \bullet [(0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1)] \\ (d) \ [(0.4; 0.5; 0.2) \times (0.2; 0.4; 0.4)] \bullet [(0.5; 0.5) \times (0.3; 0.7) \times (0.8); 0.2)] \\ (e) \ [(0.2; 0.5; 0.2; 0.2) \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.1)] \bullet [(1) \times (1) \times (1) \times (1)] \} 
 \begin{array}{l} \max \left\{ \left. (\mathbf{a}) \left[ (1) \times (1) \times (1) \right] \bullet \left[ (0.3; 0.5; 0.2) \times (0.2; 0.4; 0.4) \right] \times (0.6; 0.2; 0.2) \right] \\ (b) \left[ (0.5; 0.5) \times (0.3; 0.7) \times (0.8); 0.2) \right] \bullet \left[ (0.4, 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1) \right] \\ (c) \left[ (0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1) \right] \bullet \left[ (0.5; 0.5) \times (0.3; 0.7) \times (0.8); 0.2) \\ (d) \left[ (0.5; 0.2) \times (0.2; 0.4; 0.4) \right] \times (0.6; 0.2; 0.2) \right] \bullet \left[ (1) \times (1) \times (1) \right] \right\} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          (b) [(0,15; 0,4; 0,35; 0,1) \times (0,06; 0,26; 0,4; 0,28) \times (0,48; 0,28; 0,2; 0,04)]
(c) [(0,16; 0,4; 0,33; 0,1; 0,01) \times (0,09; 0,3; 0,37; 0,2, 0,04) \times (0,49; 0,28; 0,18; 0,04; 0,01)]
(d) [(0,15; 0,4; 0,35; 0,1) \times (0,06; 0,26; 0,4; 0,28) \times (0,48; 0,28; 0,2; 0,04)]
(e) [(0,2; 0,5; 0,2; 0,1) \times (0,1; 0,3; 0,4; 0,2) \times (0,5; 0,2; 0,01)]
                                                                                                                                                                                                                                                                                                                                             \max \left. \left\{ \begin{array}{ll} (3) \left[ 0.3; 0.5; 0.2) \times (0.2; 0.4; 0.4) \right] \times (0.6; 0.2; 0.2) \right] \\ (b) \left[ (0.15; 0.4; 0.35; 0.1) \times (0.06; 0.26; 0.4; 0.28) \times 0.48; 0.28; 0.2; 0.04) \right] \\ (c) \left[ (0.15; 0.4; 0.35; 0.1) \times (0.06; 0.26; 0.4; 0.28) \times 0.48; 0.28; 0.2; 0.04) \right] \\ (d) \left[ (0.3; 0.4; 0.35; 0.2) \times (0.2; 0.4) \times (0.4) \times (0.2; 0.4) \times (0.2; 0.4) \right] \\ \end{array} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           [(0,15;\,0,4;\,0,35;\,0,1)\times(0,06;\,0,26;\,0,4;\,0,28)\times(0,48;\,0,28;\,0,2,\,0,04)]\;\}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           [(0,15;0,4;0,35;0,1)\times(0,06;0,26;0,4;0,28)\times0,48;0,28;0,2;0,04)]\}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   max { (a) [(0,2;0,5;0,2;0,1) \times (0,1;0,3;0,4;0,2) \times (0,5;0,2;0,2;0,1)]
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           G_2*(4) = \{\ [(0.2;\,0.5;\,0.2;\,0.1)\times(0.1;\,0.3;\,0.4;\,0.2)\times(0.5;\,0.2;\,0.2;\,0.2).
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             we have
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            G_2*(3) = \ \{\ [0,3;\,0,5;\,0,2)\times(0,2;\,0,4;\,0,4)]\times(0,6;\,0,2;\,0,2)]
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             b = d
               Ш
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             a = e
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                \begin{array}{c} E_2(5,1) \bullet G_3*(4) \\ E_2(5,2) \bullet G_3*(3) \\ E_2(5,3) \bullet G_3*(2) \\ E_2(5,4) \bullet G_3*(1) \end{array}
                                                                                       F_2(3, 1) \bullet G_3*(2)

F_2(3, 2) \bullet G_3*(1)
                                                                                                                                                                                                                                                                     F_2(3,3) \bullet G_3*(0)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               F_2(4, 2) \bullet G_3*(2)

F_2(4, 3) \bullet G_3*(1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     F_2(4,4) \bullet G_3*(0)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        F_2(4, 1) \bullet G_3*(3)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         G_2*(5) = \max \{ F_2(5, 0) \bullet G_3*(5) 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         G_2*(4) = \max \{ F_2(4, 0) \bullet G_3*(4) \}
       G_2*(3) = \max \{ F_2(3, 0) \bullet G_3*(3) \}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    F_2(5,5) \bullet G_3*(0)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              As a = d and b = c, we have:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             As b (SSD, SSD, SSD) c,
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x_2^*(6) = 0 \text{ or } x_2^*(6) = 1 \text{ or } x_2^*(6) = 2 \text{ or } x_2^*(6) = 4 \text{ or } x_2^*(6) = 5 \text{ or } x_2^*(6) = 6
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   II
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                П
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  \max \left\{ (\mathbf{a}) \left[ (1) \times (1) \times (1) \right] \bullet \left[ [0.1; 0.3; 0.3; 0.3; 0.3; 0.3) \times (0; 0.1; 0.4; 0.5) \times (0.3; 0.3; 0.2; 0.2) \right] \\ (b) \left[ (1) \times (1) \times (1) \right] \bullet \left[ [0.1; 0.3; 0.3; 0.3; 0.1) \times (0.03; 0.13; 0.26; 0.37, 0.21) \times (0.32; 0.24; 0.2; 0.24; 0.2; 0.04) \right] \\ (c) \left[ (1) \times (1) \times (1) \right] \bullet \left[ [0.08; 0.3, 0.35; 0.19; 0.07; 0.01) \times (0.35; 0.23; 0.23; 0.13; 0.04, 0.01) \right] \\ (d) \left[ (0.5; 0.5) \times (0.3; 0.7) \times (0.8); 0.2) \right] \bullet \left\{ \left[ (0.1; 0.3; 0.2; 0.1) \times (0.03; 0.16; 0.33; 0.34; 0.14) \times (0.32; 0.24; 0.2; 0.04) \right] \right\} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 (b) [(0,5,0,5) \times (0,3,0,7) \times (0,8,0,2)] \bullet [(0,2,0,4,0,2;0,2) \times (0,1,0,2;0,3,0,4) \times (0,4,0,2;0,2;0,2)]
(c) [(0,4,0,5;0,1) \times (0,3;0,5;0,2) \times (0,7;0,2;0,1)] \bullet [(0,2;0,5;0,2) \times (0,1;0,3;0,4) \times (0,1;0,3;0,4,0,2) \times (0,5;0,2;0,1)]
(d) [(0,3;0,2) \times (0,2;0,4;0,4)] \bullet [(0,3;0,5;0,2) \times (0,2;0,4;0,4)] \bullet [(0,4;0,5;0,2) \times (0,2;0,4;0,4)]
(e) [(0,2;0,5;0,2;0,1) \times (0,1;0,3;0,4;0,2) \times (0,5;0,2;0,1)] \bullet [(0,4;0,5;0,1) \times (0,3;0,5;0,2) \times (0,7;0,2;0,1)]
(f) [(0,2;0,4;0,2;0,2) \times (0,1;0,2;0,3;0,4) \times (0,4;0,2;0,2;0,2)] \bullet [(0,5;0,5) \times (0,3;0,3;0,2) \times (0,8;0,2)]
(g) [(0,1;0,3;0,3) \times (0,0,1;0,4;0,5) \times (0,3;0,3;0,2,0,2)] \bullet [(1) \times (1) \times (1) \times (1)]
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       x_2(5) = 1 \text{ or } x_2(5) = 4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    \max \; \{ \, (a) \; [(1) \times (1) \times (1)] \bullet [0,1;0,3;0,3;0,3) \times (0;0,1;0,4;0,5) \times (0,3;0,2;0,2)] \}
 \begin{array}{l} [(0.2; 0.4; 0.2; 0.2) \times (0.1; 0.2; 0.3; 0.4) \times (0.4; 0.2; 0.2; 0.2; 0.2)] \\ [(0.1; 0.3; 0.3; 0.2; 0.1) \times (0.03, 0.16; 0.33; 0.34; 0.14) \times (0.32; 0.24; 0.2; 0.2; 0.04)] \\ [(0.12; 0.35; 0.35; 0.36; 0.15; 0.02) \times (0.06; 0.22; 0.36; 0.28; 0.08) \times (0.42; 0.26; 0.24; 0.06; 0.02)] \\ [(0.12; 0.35; 0.36; 0.15; 0.02) \times (0.06; 0.22; 0.36; 0.28; 0.08) \times (0.42; 0.26; 0.24; 0.06; 0.02)] \\ [(0.12; 0.35; 0.35; 0.36; 0.15; 0.02) \times (0.06; 0.22; 0.36; 0.28; 0.08) \times (0.42; 0.26; 0.24; 0.06; 0.02)] \\ [(0.1; 0.3; 0.3; 0.2; 0.1) \times (0.03, 0.16; 0.33; 0.34; 0.14) \times (0.32; 0.24; 0.25; 0.04)] \end{array} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              G_1*(y_1) = \max \{ F_1(y_1, x_1) \bullet G_2*(y_2 - x_2) : x_1 \in X_1(y_1) \}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               \begin{array}{l} (d) \ \overline{[(0.04; 0.3; 0.37; 0.2; 0.04) \times (0.04; 0.16; 0.32; 0.32; 0.16) \times (0.36; 0.24; 0.28; 0.08; 0.04)} \\ (e) \ \overline{[(0.08; 0.3, 0.35; 0.19; 0.07; 0.01) \times (0.35; 0.24; 0.23; 0.13, 0.04, 0.01)]} \\ (f) \ \overline{[(0.1; 0.3; 0.3; 0.1) \times (0.03; 0.13; 0.26; 0.37, 0.21) \times (0.32; 0.24; 0.25; 0.24; 0.2; 0.04)]} \\ (g) \ \overline{[(0.1; 0.3; 0.3; 0.3) \times (0; 0.1; 0.4; 0.5) \times (0.3; 0.3; 0.3; 0.2; 0.2)]} \end{array}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  (b) \left[ (0.1; 0.3; 0.3; 0.3; 0.2; 0.1) \times (0.03; 0.13; 0.26; 0.37, 0.21) \times (0.32; 0.24; 0.2; 0.2; 0.04) \right]
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   G_2*(5) = \{ [(0,1;0,3;0,3;0,2;0,1) \times (0,03,0,16;0,33;0,34;0,14) \times (0,32;0,24;0,2;0,2;0,04)] \}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        \underbrace{[(0.1; 0.3; 0.3; 0.2; 0.1) \times (0.03; 0.13; 0.26; 0.37, 0.21) \times (0.32; 0.24; 0.2; 0.04)}_{\{(0.08; 0.3, 0.35; 0.19; 0.07; 0.01) \times (0.35; 0.24; 0.23; 0.13; 0.04, 0.01)]\} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        (c) [(0,08;0,3,0,35;0,19;0,07;0,01) \times (0,35;0,24;0,23;0,13,0,04,0,01)]
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              [(0,2;0,4;0,2;0,2)\times(0,1;0,2;0,3;0,4)\times(0,4;0,2;0,2;0,2)]
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     c = d, we have
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   we have:
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$ \begin{aligned} &(0, [(04; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1)] \bullet [(0.2; 0.5; 0.2; 0.1) \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.2; 0.1) \\ &(0, [(0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1)] \bullet [(0.15; 0.4; 0.35; 0.1) \times (0.06; 0.26; 0.4; 0.28) \times (0.48; 0.28; 0.2; 0.04)] \\ &(\mathbf{g}, [0.3; 0.5; 0.2) \times (0.2; 0.4; 0.4)] \times (0.6; 0.2; 0.2)] \bullet [(0.15; 0.4; 0.2) \times (0.2; 0.4; 0.4)] \times (0.6; 0.2; 0.2; 0.2)] \\ &(\mathbf{h}, [0.3; 0.5; 0.2) \times (0.2; 0.4; 0.4)] \times (0.6; 0.2; 0.2)] \bullet [(0.15; 0.4; 0.35; 0.1) \times (0.6; 0.26; 0.4; 0.28) \times 0.48; 0.28; 0.2; 0.040] \\ &(\mathbf{h}, [0.2; 0.5; 0.2; 0.1) \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.2)] \bullet [(0.5; 0.5; 0.5; 0.5) \times (0.9; 0.42; 0.49) \times (0.64; 0.32; 0.04)] \\ &(\mathbf{h}, [0.2; 0.4; 0.2; 0.2) \times (0.1; 0.4; 0.2) \times (0.4; 0.2; 0.2)] \bullet [(0.5; 0.5) \times (0.3; 0.7) \times (0.8); 0.2)] \\ &(\mathbf{h}, [0.1; 0.3; 0.3; 0.3) \times (0; 0.1; 0.4; 0.2) \times (0.3; 0.2)] \bullet [(1) \times (1) \times (1)] \end{cases}$			13, 0,04, 0,01)] 5; 0,212, 0,136; 0,004, 0,004)] 3; 0,04, 0,01) 292; 0,244; 0,096; 0,028), 0,004)] 292; 0,244; 0,096; 0,028), 0,004)] 5, 0,212, 0,136, 0,04, 0,004)]	i (FSD, FSD, SSD) c, i (FSD, FSD, SSD) e d (FSD, FSD, SSD) g, d = i, f = h we have:	$x_3^*(6) = 0 \text{ or } x_3^*(6) = 1 \text{ or } x_3^*(6) = 4 \text{ or } x_2^*(6) = 5 \text{ or } x_3^*(6) = 6.$ 212, 0,136; 0,004, 0,004)]}		(6, 1, 5) (4, 1, 1) (1, 4, 1) (1, 1, 4)
(e) $[(0,4;0,5;0,1) \times (0,3;0,5;0,2) \times (0,7;0,2;0,1)]$ (f) $[(0,4;0,5;0,1) \times (0,3;0,5;0,2) \times (0,7;0,2;0,1)]$	(g) $[0,3;0,5;0,2)\times(0,2;0,4;0,4)]\times(0,6;0,2;0,2)$ (h) $[0,3;0,5;0,2)\times(0,2;0,4;0,4)]\times(0,6;0,2;0,2)$	(i) $[(0.2; 0.5; 0.1) \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.2; 0.1)] \bullet [0.25; 0.5; 0.25; 0.15]$ (j) $[(0.2; 0.4; 0.2; 0.2) \times (0.1; 0.2; 0.3; 0.4) \times (0.4; 0.2; 0.2; 0.2)] \bullet [(0.5; 0.5) \times (0.5; 0.5)]$ (k) $[0.1; 0.3; 0.3; 0.3) \times (0; 0.1; 0.4; 0.5) \times (0.3; 0.3; 0.2; 0.2)] \bullet [(1) \times (1) \times (1)]$	$ \begin{aligned} &(a) \left[(0.1; 0.3; 0.3; 0.3) \times (0; 0.1; 0.4; 0.5) \times (0.3; 0.2; 0.21) \times (0.3; 0.3; 0.24; 0.2; 0.04) \\ &(b) \left[(0.1; 0.3; 0.3; 0.3; 0.1) \times (0.03; 0.13; 0.26; 0.37, 0.21) \times (0.32; 0.24; 0.2; 0.04) \\ &(c) \left[(0.08; 0.3, 0.35; 0.19; 0.07; 0.01) \times (0.03; 0.14; 0.29; 0.32; 0.18; 0.04) \times (0.35; 0.24; 0.23; 0.13, 0.04, 0.01) \right] \\ &(d) \left[(0.05; 0.225; 0.35; 0.19; 0.07; 0.01) \times (0.03; 0.14; 0.29; 0.32; 0.18; 0.04) \times (0.35; 0.28; 0.212, 0.136; 0.004, 0.004) \right] \\ &(e) \left[(0.06; 0.225; 0.35; 0.19; 0.07; 0.01) \times (0.03; 0.14; 0.29; 0.32; 0.18; 0.04) \times (0.35; 0.23; 0.23; 0.13; 0.04, 0.01) \right] \\ &(e) \left[(0.06; 0.235; 0.355; 0.255; 0.085; 0.01) \times (0.18; 0.108; 0.16) \times (0.356; 0.22; 0.058; 0.04) \right] \\ &(e) \left[(0.06; 0.235; 0.355; 0.255; 0.085; 0.01) \times (0.18; 0.108; 0.16) \times (0.36; 0.24; 0.028; 0.04) \right] \\ &(e) \left[(0.06; 0.235; 0.355; 0.255; 0.085; 0.01) \times (0.18; 0.108; 0.202; 0.038; 0.029; 0.038; 0.04) \right] \\ &(e) \left[(0.05; 0.225; 0.355; 0.255; 0.085; 0.01) \times (0.18; 0.108; 0.202; 0.038; 0.029; 0.038; 0.004) \right] \\ &(e) \left[(0.05; 0.225; 0.355; 0.255; 0.085; 0.01) \times (0.18; 0.108; 0.221; 0.336; 0.22; 0.058; 0.04) \right] \\ &(e) \left[(0.05; 0.225; 0.355; 0.255; 0.085; 0.01) \times (0.18; 0.108; 0.221; 0.235; 0.235; 0.038; 0.01) \times (0.18; 0.108; 0.221; 0.038; 0.22; 0.058; 0.04) \right] \\ &(e) \left[(0.05; 0.225; 0.355; 0.255; 0.085; 0.01) \times (0.18; 0.108; 0.221; 0.244; 0.25; 0.048; 0.021; 0.136; 0.044; 0.044; 0.136; 0.221; 0.044; 0.035; 0.221; 0.044; 0.035; 0.221; 0.044; 0.035; 0.221; 0.044; 0.035; 0.221; 0.044; 0.035; 0.221; 0.044; 0.035; 0.221; 0.043; 0.0221; 0.044; 0.035; 0.221; 0.044; 0.035; 0.221; 0.044; 0.035; 0.225; 0.235; 0.2$	o) c d (ASD, FSD, SSD) e b = j, c = e	$ \begin{aligned} & \text{[0,1; 0,3; 0,3; 0,3; 0,3; 0,3; 0,1; 0,4; 0,5)} \\ & \text{[10,1; 0,3; 0,3; 0,3; 0,3; 0,1)} \times & \text{(0,0,1; 0,4; 0,5)} \\ & \text{(0,0,1; 0,3; 0,3; 0,3; 0,1)} \times & \text{(0,0,0,1; 0,3; 0,2)} \times & \text{(0,0,0,0,0,0)} \\ & \text{[(0,0,0,0,0,2,5; 0,35; 0,25; 0,1; 0,025)} \times & \text{(0,009; 0,069; 0,211, 0,333, 0,28, 0,098)} \times & \text{(0,32; 0,288; 0,212, 0,136; 0,004, 0,004)} \\ & \text{(0,05; 0,225; 0,35; 0,25; 0,1; 0,025)} \times & \text{(0,009; 0,069; 0,211, 0,333, 0,28, 0,098)} \times & \text{(0,32; 0,288; 0,212, 0,136; 0,004, 0,004)} \\ & \text{(0,05; 0,225; 0,35; 0,25; 0,1; 0,025)} \times & \text{(0,009; 0,069; 0,211, 0,333, 0,28, 0,098)} \times & \text{(0,32; 0,288; 0,212, 0,136; 0,004, 0,004)} \\ & \text{(0,05; 0,225; 0,35; 0,25; 0,1; 0,025)} \times & \text{(0,009; 0,069; 0,211, 0,333, 0,28, 0,098)} \times & \text{(0,32; 0,288; 0,212, 0,136; 0,004, 0,004)} \\ & \text{(0,05; 0,225; 0,35; 0,25; 0,1; 0,025)} \times & \text{(0,009; 0,069; 0,211, 0,333, 0,28, 0,098)} \times & \text{(0,32; 0,288; 0,212, 0,136; 0,004, 0,004)} \\ & \text{(0,05; 0,225; 0,35; 0,25; 0,1; 0,025)} \times & \text{(0,009; 0,069; 0,211, 0,333, 0,28, 0,098)} \times & \text{(0,32; 0,288; 0,212, 0,136; 0,004, 0,004)} \\ & \text{(0,05; 0,225; 0,35; 0,25; 0,1; 0,025)} \times & \text{(0,009; 0,069; 0,211, 0,333, 0,28, 0,098)} \times & (0,32; 0,288; 0,212, 0,288; $		(5, 0, 1) (1, 5, 0) (1, 0, 5) (0, 5, 1)
$F_1(6, 2) \bullet G_2*(4)$	$F_1(6,3) \bullet G_2^*(3)$	F_1 (6, 4) \bullet $G_2*(2)$ F_1 (6, 5) \bullet $G_2*(1)$ F_1 (6, 6) \bullet $G_2*(0)$ }	= max { (a) [0,1; 0,3; 0,3; 0,3) × (0; 0,1; 0,4; 0,1; 0,4; 0,1; 0,3; 0,3; 0,3; 0,3; 0,3; 0,3; 0,3; 0,3	As c(FSD, FSD, FSD) g, e (FSD, FSD) g, d (FSD, FSD, SSD) c I (FSD, FSD, SSD) g, d (FSD, FSD, FSD) f i (FSD, FSD, FSD) f, a = k,	$\begin{split} G_{o}*(6) &= \{ [0,1;0,3;0,3;0,3) \times (0;0,1;0,4;0,5) \times (0,3;0,3;0,2;0,2) \} \\ &= [(0,1;0,3;0,3;0,2;0,1) \times (0,03;0,13;0,26;0,37,0,21) \times (0,03;0,3;0,3;0,3;0,35;0,35;0,35;0,35;0,3$	The non-dominated realizations of the process are as follows: $(y_1 = 6, x_1 = 6, y_2 = 0, x_2 = 0, y_3 = 0, x_3 = 0)$ $(y_1 = 6, x_1 = 0, y_2 = 6, x_2 = 6; y_3 = 0, x_3 = 0)$ $(y_1 = 6, x_1 = 0; y_2 = 6; x_2 = 6; y_3 = 6; x_3 = 6)$ $(y_1 = 6, x_1 = 5; y_2 = 1, x_2 = 1, y_3 = 0, x_3 = 0)$	The non-dominated allocations are as follows: (6, 0, 0) (0, 6, 0); (0, 0, 6); (3, 1, 0);

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A COMPREHENSIVE EVALUATION OF SUSTAINABLE MANUFACTURING PROGRAMS USING ANALYTIC NETWORK PROCESS (ANP)

Abstract

Evaluation of programs initiated by manufacturing firms that are geared toward sustainability is worthy of attention in research due to the current global demands of addressing not just economic growth but environmental and social burdens. This paper attempts to provide a comprehensive evaluation framework using the hierarchical structure of sustainable manufacturing (SM) indicators set developed by the US National Institute of Standards and Technology (US NIST) and a multi-criteria decisionmaking (MCDM) approach, the analytic network process (ANP). ANP is deemed appropriate, aside from the multi-criteria nature of the problem, because of the presence of subjective components that are interrelating in complex relationships. A real case study is carried out in a semiconductor manufacturing firm in the Philippines in the evaluation of its programs toward sustainability. The results show that the creation and implementation of cleaner production technologies are considered the most relevant programs. Developing energy-efficient products and adopting lean six sigma programs are considered second on the list. This paper proposes that sustainability is achieved by formulating strategies that enhance customer and community well-being via addressing environmental concerns especially on toxic substance, greenhouse gas (GHG) and air emissions. The contribution of this paper consists in providing an evaluation framework which is comprehensive enough to capture real-life complex decision-making processes. Limitations and possibilities for future research are also presented in this paper.

Keywords: Multi-criteria decision-making, sustainable manufacturing, sustainability, analytic network process.

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1. Introduction

Due to several emerging concerns about sustainable development, manufacturing firms are compelled by various stakeholders that require firms to align their manufacturing processes and manufactured products along with the context of sustainability. This issue raises various questions that force researchers and practitioners to discuss matters in different areas of the sustainability domain. Towards this end, a widely-accepted approach is to use the concept of the triplebottom line (TBL) (Elkington, 1997). TBL expanded traditional, purely profitbased strategies into initiatives that address environmental, economic and social issues. A parallel and recently-organized focus under the sustainability umbrella is the sustainable manufacturing (SM) approach which is defined by the US Department of Commerce as the "creation of manufactured products that use processes that minimize negative environmental impact, conserve energy and natural resources, are safe for employees, communities and consumers, and are economically sound" (International Trade Administration, 2007; Joung et al., 2013). Studies in the literature converged on the idea which suggests that firms that promote sustainability as their focus are more likely to be successful in their respective industries (Azapagic, 2003; Jayal et al., 2010).

While the motivation of SM is clear, the approaches that would link these elusive concepts to manufacturing decisions remain vague. Discussion of the current literature focuses on how to refine these concepts of SM to a plausible level of being concrete and operational (Labuschagne et al., 2005). Ocampo and Clark (2014a) found out that current strategies of manufacturing firms are fragmented in the sustainability focus which may result in unorganized and illdirected utilization of company resources. With several approaches and initiatives published in the literature, leaning toward addressing sustainability issues, such as cleaner production, corporate social responsibility (CSR), eco-efficiency (Lozano, 2012), life-cycle assessment (Ageron et al., 2012), ISO certifications (Lozano, 2012; Ageron et al., 2012), manufacturing firms are left with a challenge of determining the priorities attached to each initiative in relation to SM. Such evaluation of these approaches is deemed necessary to elucidate their significance on sustainability and thus providing firms with relevant information for decision-making. Due to the complexity of such evaluation involving tangible and highly intangible aspects with assessment structure that comprises value judgments, assumptions and scenarios (Heijungs et al., 2010), a multi-criteria decision-making (MCDM) approach is deemed appropriate (Cho, 2003; Herva and Roca, 2013). For instance, the evaluation of CSR activities such as company involvement in community-enhancement projects would require measurement framework that is hardly quantifiable because of the presence of factors with no available measurement system.

Previous studies have embarked on MCDM methods in environmental or sustainability assessment. These methods include analytic hierarchy process (AHP) (de Brucker et al., 2013), analytic network process (ANP) (Tseng et al., 2009a), fuzzy set theory (Tseng et al., 2009b), preference ranking organization method for enrichment evaluation (PROMETHEE) (Vinodh and Girubha, 2012), grey system theory (Baskaran et al., 2012) and decision-making trial and evaluation laboratory (DEMATEL) (Tseng et al., 2012). Aside from being a multi-criteria problem, evaluation of SM programs must reflect interdependencies and interrelationships of decision components which are inherent in the sustainability framework (Ocampo and Clark, 2014a). Considering such an argument, ANP is used in this study because of the following reasons: (1) sustainability program evaluation is a complex and multi-dimensional problem which characterizes the ANP framework; and (2) ANP overcomes hierarchical limitation, as most of MCDM methods have, and supports interrelationships of decision components (Saaty, 2001). Although various works have been published on sustainability assessment, a comprehensive evaluation of the most relevant SM program at firm level is missing in the literature. This area is significant as it provides valuable insights for managers and decision-makers in manufacturing firms especially on selecting programs in the presence of tangible and intangible criteria in addition to the inherent interrelationships among decision components. Thus, the objective of this paper is to present an evaluation method for selecting the most relevant SM program in the context of comprehensive consideration of the TBL. An evaluation system based on the US National Institute of Standards and Technology (US NIST) is presented in this paper and a case study of a semiconductor manufacturing firm is used to convey the methodology.

This paper is organized as follows. Section 2 provides a review of the literature in sustainability evaluation framework and a review of the ANP. Section 3 presents the general methodology of the evaluation problem. Section 4 presents a case study in a semiconductor manufacturing firm. Section 5 shows the results of such evaluation using ANP. Section 6 provides a discussion of the relevance of the results to sustainability assessments. Section 7 concludes the study with a short discussion of future research.

2. Literature review

2.1. Approaches to sustainability evaluation

Current approaches in this area are focused on developing sustainability indicators. Indicators provide standards in evaluating products, processes, companies, economic sectors or even countries in view of SM (Joung et al., 2013). A number of indicator sets are known in the literature from various sources such as the

government, private sector, research and academic institutions. Among these indicator sets are the Global Report Initiative (GRI, 2006), the Dow Jones Sustainability Indexes (SAM Index, 2007), the Institution of Chemical Engineers Sustainability Metrics (IChemE, 2002), United Nations-Indicators of Sustainable Development (UN CSD, 2007), the Wuppertal Sustainability Indicators (Spangenberg and Bonniot, 2007), the 2005 Environmental Sustainability Indicators (ESI, 2005), the European Environmental Agency Core Set of Indicators (EEA CSI, 2005), the Environmental Performance Index (EpfI, 2010), the Organization for Economic Cooperation and Development Core Environmental Indicators (OECD CEI, 2003), the Japan National Institute of Science and Technology Policy (JSTA, 1995), the Ford Product Sustainability Index (Schmidt and Taylor, 2006), the Environmental Pressure Indicators for European Union (EprI, 1999), the General Motors Metrics for Sustainable Manufacturing (Feng et al., 2010; Dreher et al., 2009), the Wal-Mart Sustainability Product Index (Walmart Sustainability Product Index, 2009) and the International Organization for Standardization Environment Performance Evaluation Standard (ISO, 1999). The challenge of these indicators lies both in comprehensiveness and in being operational. Joung et al. (2013) developed a systematic integration of 11 indicator sets (see Joung et al., 2013). The resulting integration was formed into a hierarchical structure of an SM indicator set. This interesting work outlined a more comprehensive and operational SM because the integrated indicator set came from a number of established indicator sets. Furthermore, due to its hierarchical structure, the details of remembering decision components are more defined as one goes down the hierarchy.

Another stream of current research in this domain supports measuring sustainability performance of a product or manufacturing facility. De Silva et al. (2009) developed a scoring method for product sustainability index from a TBL approach. Ghadimi et al. (2012) proposed a sustainability product assessment methodology. Jaafar et al. (2007) presented a comprehensive procedure for computing PSI by calculating the weighted sum of different subelements within the triple-bottom line for each life-cycle stage (pre-manufacturing, manufacturing, use and post-use). A hierarchical approach using AHP with time element in evaluating the sustainable development index of firms was proposed by Krajnc and Glavic (2005). However, none of these studies deals with the selection of an SM program in a comprehensive TBL-based evaluation framework.

2.2. Analytic network process (ANP)

Analytic Hierarchy Process (AHP)/Analytic Network Process (ANP), developed by Saaty (1980; 2001) is a general theory of relative measurement. It is used to derive priority scales from paired comparisons of elements with respect to a higher element in the hierarchy or network. Comparisons are taken from actual

measurements using Saaty's fundamental scale of absolute numbers. Elements are organized into homogeneous clusters or components in the decision problem. AHP/ANP is used in almost all applications related to decision-making such as planning, selection of the best alternative, conflict resolution, resource allocation and even optimization (Cho, 2003; Chen et al., 2012; Ocampo and Clark, 2014b). ANP, which is a generalization of AHP, organizes decision models represented in networks of decision components of alternatives, criteria, objectives and other factors that influence each other's priorities (Cho, 2003). This allows for flexibility in decision-making, taking all consideration of interdependencies and interrelationships among decision components and elements which are often representative of actual real-life scenarios. A special case where decision components are organized in a multi-level hierarchy is an AHP.

Local priorities in ANP are computed in a way similar to how local priorities in AHP are established, based on pairwise comparisons and judgments. The advantage of using ANP in a wide array of decision problems lies in capturing both qualitative and quantitative criteria in an analytical decision model and then allowing interrelationships with each decision component, which is one of the shortcomings of current unilateral decision-making schemes. Figure 1 represents an example of the form of the network structure used in a particular decision-making process. A comprehensive theoretical discussion of networks, interdependencies, pairwise comparisons processes along with the framework of AHP/ANP can be found in Saaty (1980; 2001).

The input to the supermatrix of a hierarchical network depends on the presence and type of dependence relations described in the digraph as shown in Figure 1.

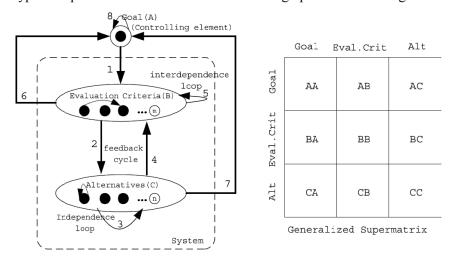


Figure 1. A sample network structure and its corresponding supermatrix

Source: Adapted from: Promentilla et al. (2006a).

The values of the block matrices, for instance AB, in the initial supermatrix are the estimated priorities that provide the relative strength of dominance of an element over another element in the component with respect to a common element from which the arc emanates. The eigenvector method is one of the popular methods used to quantify the relative dominance of the elements from pairwise comparison matrices. Saaty (1980) proposed the following eigenvalue formulation to obtain the desired ratio-scale priority vector (or weights) w of n elements:

$$\mathbf{A}\mathbf{w} = \lambda_{\text{max}}\mathbf{w} \tag{1}$$

where **A** is the positive reciprocal pairwise comparisons matrix, λ_{max} is the maximum (or principal) eigenvalue of the matrix **A**.

The measure of consistency of judgment is based on using the Consistency Index (CI) and Consistency Ratio (CR). The Consistency Index (CI), as a measure of degree of consistency, was calculated using the formula:

$$CI = \frac{\lambda_{\text{max}} - n}{n - 1} \tag{2}$$

The consistency ratio (CR) is computed as:

$$CR = \frac{CI}{RI}$$
 (3)

where RI is the mean random consistency index [see Alonso and Lamata (2006) for Tables of RI]. Acceptable CR values must be less than 0.1. Decision-makers were asked to repeat the pairwise comparisons for CR values greater than 0.1.

Global priority ratio scales or priorities can be obtained based on the synthesizing concept of the supermatrix. By raising the matrix to large powers, the transmission of influence along all possible paths defined in the decision structure is captured in the process (Saaty, 2001). The convergence of initial priorities (stochastic matrix) to an equilibrium value in the limit supermatrix provides a set of meaningful synthesized priorities from the underlying decision structure (Promentilla et al., 2008). Saaty (2001) assured that as long as the supermatrix representation is a primitive irreducible matrix in a strongly connected digraph, the initial supermatrix must converge to a limit supermatrix. Promentilla et al. (2008) discussed that the limit supermatrix denoted by L exists when the initial supermatrix is standardized by its principal eigenvalue as shown by the equation:

lim_{p→∞}
$$\left(\frac{\mathbf{S}}{\lambda_{\text{max}}}\right)^p = \lim_{p\to\infty} \left(\overline{\mathbf{S}}\right)^p = \mathbf{L}$$
 (4)

Each column of the limit supermatrix is a unique positive column eigenvector associated with the principal eigenvalue λ_{max} (Promentilla et al., 2008). This principal column eigenvector corresponds to the stable priorities from the limit supermatrix and can be used to measure the overall relative dominance of one element over another in a hierarchical network structure (Promentilla et al., 2006a).

Table 1

3. Methodology

In general, the proposed procedure in evaluating SM programs is as follows:

- 1. Incorporate feedback and dependence on the hierarchical SM structure which was organized by Joung et al. (2013) and is published on the US NIST website (SMIR, 2011). The details of each component can be accessed through the website. Introducing interdependencies is done by gradually introducing feedback and dependence loops to the hierarchical SM structure. A group of experts must establish these loops based on theoretical and practical perspectives of sustainability. The general evaluation network is shown in Figure 2. Note that if an arrow emanates from C1 to C2 in the decision network, it means that C1 is influenced by C2. An arrow emanating from and to the same element or component means an existence of inner dependence of an element or elements within a component.
- 2. Elicit pairwise comparisons based on the network developed in 1. In eliciting paired comparisons, in general we ask the question: "Given a control element, a component (element) of a given network, and given a pair of components (or elements), how much more does a given member of the pair dominate the other member of the pair with respect to a control element?" (Promentilla et al., 2006a). Saaty's Fundamental Scale (1980) is used to compare elements pairwise as shown in Table 1. A pairwise comparison matrix has a reciprocal characteristic. For instance, comparing a₁ with a₂ will have a 3 ratio scale, then comparing a₂ with a₁ should have a ratio scale of 1/3 as the reciprocal of 3. Local priority vectors are obtained using equation (1). Consistency ratios are checked using equations (2) and (3).

Saaty's Fundamental Scale

	Definition	Explanation
1	Equal importance	Two elements contribute equally to the objective
2	Weak	between equal and moderate
3	Moderate importance	Experience and judgment slightly favor one element over another
4	Moderate plus	between moderate and strong
5	Strong importance	Experience and judgment strongly favor one element over another
6	Strong plus	between strong and very strong
7	Very strong or demonstrated	An element is favored very strongly over another; its dominance
	importance	demonstrated in practice
8	Very, very strong	between very strong and extreme
9	Extreme importance	Evidence favoring one element over another is either of the highest possible order or affirmation

3. Populate the initial supermatrix with the local priority vectors obtained in step 2. Then transform the initial supermatrix to a column stochastic matrix by normalizing column values so that sum of each column is equal to 1. This is done by dividing each value in a column by the sum of that column. Finally, using equation 4, raise the stochastic supermatrix to sufficiently large powers until each column becomes identical. The resulting values are the principal vector of dominance of the elements in the supermatrix.

4. Case study

To illustrate the methodology, a real case study is carried out in a semiconductor manufacturing firm in the Philippines. The profile of the firm and the SM programs undertaken have been published elsewhere (Ocampo and Clark, 2014a). FC semiconductor, being a multinational firm, has manufacturing sites strategically located in Asia with a test and assembly site in Cebu, Philippines (Ocampo and Clark, 2014a). The firm, is committed to incorporate sustainability in their decision-making especially in their products and processes. The firm has promoted ten programs in their approach toward sustainability. These are: reforestation program (PI), health and wellness program (P2), competitive employee compensation and career development (P3), sound occupational health and safety (P4), elimination of lead in plating process (P5), adoption of "green" molding compound (P6), elimination of PVC in plastic packaging (P7), energy efficient products (P8), lean six sigma projects (P9) and energy management program (P10). The firm is faced with the problem: to which programs they must attach higher priorities in their effort and resources to characterize sustainability effectively.

Derived from the work of Joung et al. (2013) on the comprehensive sustainability indicator set and the case information of SM programs, Table 2 shows identified clusters or decision components with their corresponding codes. Figures 3-5 elucidate the decision network based on the general framework in Figure 2. Environmental, economic and social criteria are coded with A, B, and C, respectively. The subcriteria, in the level-2 cluster, are coded in a way that shows reference from their parent criterion. For instance, the subcriteria under environmental criteria are coded as A_i , i = 1,2,3,...,n. The attributes, in the level-3 cluster, are likewise coded in a form that references their parent subcriteria. Attributes under A_1 subcriteria for instance, are coded as A_{1j} , j = 1,2,3,...,k. SM programs are coded as P_1 , 1 = 1,2,3,...,m.

Table 2

Decision components and their codes

Decision components and elements	Code	Decision components and elements	Code	Decision components and elements	Code
Evaluation of sustainable manufacturing	G	Effluent	A21	Employees health and safety	C11
Environmental stewardship	A	Air emissions	A22	Employees career development	C12
Economic growth	В	Solid waste emissions	A23	Employee satisfaction	C13
Social well-being	C	Waste energy emissions	A24	Health and safety impacts from manufacturing and product use	C21
Pollution	A1	Water consumption	A31	Customer satisfaction with operations and products	C22
Emissions	A2	Material consumption	A32	Inclusion of specific rights to customer	C23
Resource consumption	A3	Energy/electrical consumption	A33	Product responsibility	C31
Natural habitat conservation	A4	Land use	A34	Justice/equity	C32
Profit	B1	Biodiversity management	A41	Community development programs	C33
Cost	B2	Natural habitat quality	A42	Reforestation program	P1
Investment	B3	Habitat management	A43	Health and wellness program	P2
Employee	C1	Revenue	B11	Competitive employee compensation and career development	Р3
Customer	C2	Profit	B12	Sound occupational health and safety	P4
Community	C3	Materials acquisition	B21	Elimination of lead in plating process	P5
Toxic substance	A11	Production	B22	Adoption of "green" molding compound	P6
Greenhouse gas emissions	A12	Product transfer to customer	B23	Elimination of PVC in plastic packaging	P7
Ozone depletion gas emis-	A13	End-of-service-life product	B24	Energy efficient products	P8
sions		handling			
Noise	A14	Research and development	B31	Lean six sigma programs	P9
Acidification substance	A15	Community development	B32	Energy management program	P10

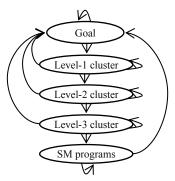


Figure 2. General evaluation framework based on ANP

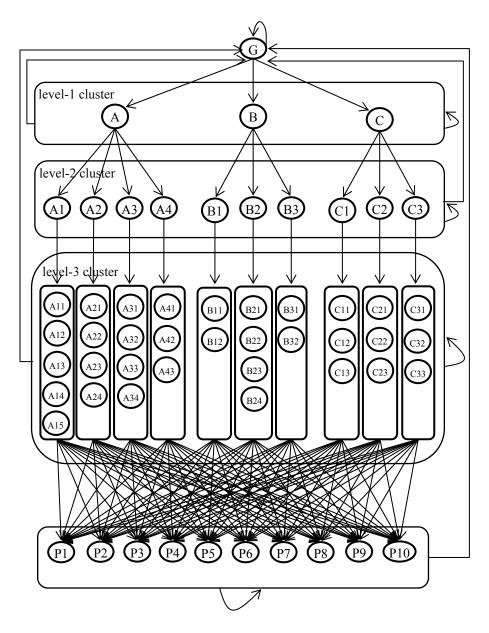


Figure 3. Decision problem of the evaluation of sustainable manufacturing programs

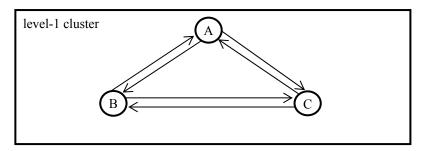


Figure 4. Interdependencies of the level-1 cluster

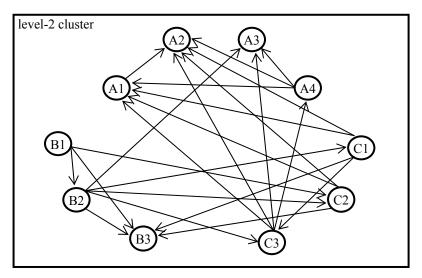


Figure 5. Interdependencies of the level-2 cluster

Note that the level-1 to level-3 clusters, as well as the SM programs cluster, have outer dependence loop to the goal. Thus, the goal serves as a controlling element of the decision network shown in Figure 3. This is consistent with the emphasis of Saaty on the existence of a control hierarchy on ANP (Saaty, 2001). In practice, this loop guarantees that the elements in the lower level clusters conform to the goal. This ensures a strong irreducible digraph which is a requisite to achieve a limit supermatrix (Promentilla 2006a; 2006b). A group of experts in sustainability and manufacturing research and practice has been invited to a focus group discussion (FGD) to provide inputs on the interdependencies of the hierarchical framework of Joung et al. (2013) and to conduct paired comparisons based from these interdependencies. The expert group is composed of four researchers and five manufacturing managers and consultants who have sufficient background in manufacturing and sustainability research. This method of gathering experts' judgments is consistent with several applications of ANP in various

domains, e.g. Promentilla et al (2006b); Tseng et al. (2009a). The group was already familiar with the purpose of the discussion and the hierarchical structure of the evaluation framework before the FGD was conducted. Based from the group's unified judgment, interdependencies of cluster-1 and cluster-2 are identified as shown in Figures 4 and 5. The results of paired comparisons are shown in the next section.

5. Results

In general, there are six types of paired comparisons in this paper. The first four sets are the results of the hierarchical dependence from the goal down to cluster-1, from cluster-1 to cluster-2, from cluster-2 to cluster-3 and from cluster-3 to the SM programs cluster, while the last two sets are drawn from the interdependencies described in Figures 4 and 5. First, paired comparisons are done on the dependence of cluster-1 elements with respect to the goal. Second, paired comparisons are done on the dependence of cluster-2 elements with respect to their parent element in the first cluster. Third, paired comparisons are done on the dependence of cluster-3 elements with respect to their parent element in the second cluster. Fourth, paired comparisons are done based on the efficiency of elements in the SM programs cluster, with respect on each element in the third cluster. Fifth, paired comparisons are done on the influence of elements on other elements in cluster-1. Lastly, paired comparisons are done on the influence of elements on other elements on other elements in cluster-2.

For the purpose of brevity, we show here only samples of paired comparisons and the general structure of the supermatrix. Due to the large space needed for a 56x56 supermatrix, we could not present here the initial, column stochastic and limiting supermatrices. Readers are advised to contact the corresponding author through email if they wish to have a Microsoft Excel file of these supermatrices. Table 3 shows a sample of the paired comparisons of the first type. The question being asked in Table 3 is: "Comparing environmental stewardship (A) and economic growth (B), which one dominates the goal (G) more and by how much?" The resulting eigenvector (priority vector) is shown in Table 3. Table 4 shows a sample of the paired comparisons of the second type. The question being asked in Table 4 is: "Comparing pollution (A1) and emission (A2), which one dominates environmental stewardship (A) more, and by how much?". Table 5 shows a sample of the paired comparisons of the third type. The question being asked in Table 5 is: "Comparing toxic substances (A11) and greenhouse gas emissions (A12), which one dominates pollution (A1) more, and by how much?". Table 6 shows a sample of pairwise comparisons matrix of the performance of SM programs with respect to each element in cluster-3. The question being asked in Table 6 is this: "Comparing reforestation program (P1) and health and wellness program (P2), which one characterizes toxic substance (A11) better and by how much?". The resulting priority vector is reported in Table 6. Table 7 shows the dominance of other elements over a specific element in cluster-1. The question in Table 7 is: "Comparing environmental stewardship (A) and economic growth (B), which one dominates environmental stewardship (A) more and by how much?". The resulting priority vector is reported in Table 7. Lastly, Table 8 also shows a sample of pairwise comparisons matrix of the interdependencies of elements on cluster-2. The question being asked in Table 8 is: "Comparing pollution (A1) and emission (A2), which one influences the community (C3) more and by how much?". The resulting priority vector is again reported.

Table 3 Pairwise comparisons of the dependence of cluster-1 elements on the goal

A	A	В	С	Eigenvector
A	1	1/2	1/2	0.200
В	2	1	1	0.400
C	2	1	1	0.400

Table 4
Pairwise comparisons of the dependence of cluster-2 elements on their parent element in cluster-1

A	A1	A2	A3	A4	Eigenvector
A1	1	1	3	2	0.349
A2	1	1	3	2	0.349
A3	1/3	1/3	1	2	0.147
A4	1/2	1/2	1/2	1	0.155

Table 5 Pairwise comparisons of the dependence of cluster-3 elements on their parent element in cluster-2

A1	A11	A12	A13	A14	A15	Eigenvector
A11	1	1	3	5	3	0.349
A12	1	1	3	5	3	0.349
A13	1/3	1/3	1	2	1	0.118
A14	1/5	1/5	1/2	1	1/2	0.066
A15	1/3	1/3	1	2	1	0.118

 $\label{eq:Table 6} Table \ 6$ Pairwise comparisons of the performance of SM programs with respect to an element in cluster-3

A11	P1	P2	Р3	P4	P5	P6	P7	P8	P9	P10	Eigenvector
P1	1	1/5	1	1/4	1/7	1/7	1/7	1/2	1	1/2	0.028
P2	5	1	4	2	1/2	1/2	1/2	3	4	3	0.115
P3	1	1/4	1	1/3	1/5	1/5	1/5	1/2	1	1/2	0.036
P4	4	1/2	3	1	1/3	1/3	1/3	2	3	2	0.074
P5	7	2	5	3	1	1	1	4	5	4	0.205
P6	7	2	5	3	1	1	1	3	5	3	0.199
P7	7	2	5	3	1	1	1	4	5	4	0.205
P8	2	1/3	2	1/2	1/4	1/3	1/4	1	2	1	0.053
P9	1	1/4	1	1/3	1/5	1/5	1/5	1/2	1	1/2	0.036
P10	2	1/3	2	1/2	1/4	1/4	1/4	1	2	1	0.049

Table 7 Pairwise comparisons of the dominance of other elements with respect to an element in cluster-1

A	A	В	С	Eigenvector
Α	1	3	2	0.545
В	1/3	1	1/2	0.168
C	1/2	2	1	0.287

Table 8 Pairwise comparisons of the dominance of criteria with respect to an element C3 in cluster-2

C3	A1	A2	A3	A4	Eigenvector
A1	1	2	4	3	0.480
A2	1/2	1	3	2	0.262
A3	1/4	1/3	1	1/2	0.103
A4	1/3	1/2	2	1	0.155

The supermatrix shown in Table 9 is populated by the priority vectors obtained from the six types of paired comparisons. To facilitate discussion, we let A, B, C, D and E denote clusters of the goal, cluster-1, cluster-2, cluster-3 and SM programs cluster, respectively. In general, based on the hiernet presented in Figure 1, the supermatrix can be structured as in Table 9.

Table 9
The general supermatrix

	A	В	C	D	E
A	1	1	1	1	1
В	BA	BB	0	0	0
C	0	diag [CB]	CC	0	0
D	0	0	diag [DC]	I	0
Е	0	0	0	DC	I

Note that the first row in the supermatrix which comprises blocks AA, AB, AC, AD, and AE is a unity vector. This represents the feedback control loop from all clusters to the goal element. Block BA (which means that B dominates A) is a hierarchical dependence from goal to cluster-1. Block CB and block DC are diagonal matrices resulting from the dominance of lower level elements to their parent criteria. CB denotes dependence of cluster-2 elements on their parent cluster-1 element while DC is the dependence of cluster-3 elements on their parent cluster-2 elements. Block BB and block CC denote interdependencies of cluster-1 and cluster-2 elements, respectively. Block DC is a hierarchical dependence of SM programs cluster on each element in cluster-3. Identity matrices which are represented by blocks DD and EE, show inner dependence of the elements on the cluster-3 and the SM programs cluster, respectively. Null matrices for the rest of the blocks in the supermatrix describe lack of feedback and dependence on the elements of decision clusters. After populating the supermatrix with the local priority vectors, a stochastic matrix is then obtained by dividing column values by the sum of that column. By applying equation 4, the column stochastic matrix is raised to large powers until it converges to its Cesaro sum. Convergence is observed if each column in the supermatrix is identical. Each column represents the principal right eigenvector of the supermatrix. Priority ranking of elements per cluster is shown in Table 10.

Table 10 Priority ranking of decision components

		Priority Vector	
	Raw	Distributive	Ideal
G	0.3958	1.0000	1.0000
В	0.1156	0.3896	1.0000
C	0.1131	0.3811	0.9782
A	0.0681	0.2293	0.5886
C2	0.0322	0.1752	1.0000
В3	0.0246	0.1338	0.7638
B2	0.0228	0.1242	0.7092
A2	0.0227	0.1234	0.7043
A1	0.0192	0.1045	0.5965
B1	0.0176	0.0959	0.5475
C1	0.0144	0.0784	0.4474
A3	0.0129	0.0704	0.4016
C3	0.0125	0.0683	0.3898
A4	0.0048	0.0260	0.1483

B32 0.0082 0.1104 1.0000 A22 0.0052 0.0705 0.6383 B31 0.0041 0.0552 0.5000 C23 0.0039 0.0520 0.4713 C22 0.0039 0.0520 0.4713 A11 0.0033 0.0451 0.4085 A12 0.0033 0.0445 0.4032 B11 0.0033 0.0445 0.4032 B12 0.0033 0.0445 0.4032 A21 0.0026 0.0352 0.3192 C11 0.0026 0.0352 0.3192 C11 0.0026 0.0349 0.3163 B21 0.0023 0.0308 0.2785 B22 0.0023 0.0308 0.2785 B22 0.0023 0.0308 0.2785 B33 0.0019 0.0261 0.2366 A34 0.0019 0.0261 0.2366 C21 0.0019 0.0260 0.2357 C31 <th></th> <th></th> <th></th> <th></th>				
B31 0.0041 0.0552 0.5000 C23 0.0039 0.0520 0.4713 C22 0.0039 0.0520 0.4713 A11 0.0033 0.0451 0.4085 A12 0.0033 0.0445 0.4032 B11 0.0033 0.0445 0.4032 B12 0.0033 0.0445 0.4032 A21 0.0026 0.0352 0.3192 A23 0.0026 0.0352 0.3192 C11 0.0026 0.0349 0.3163 B21 0.0023 0.0308 0.2785 B22 0.0023 0.0308 0.2785 B22 0.0023 0.0308 0.2785 A33 0.0019 0.0261 0.2366 A34 0.0019 0.0261 0.2366 A31 0.0019 0.0261 0.2357 C31 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 A41 <td>B32</td> <td>0.0082</td> <td>0.1104</td> <td>1.0000</td>	B32	0.0082	0.1104	1.0000
C23 0.0039 0.0520 0.4713 C22 0.0039 0.0520 0.4713 A11 0.0033 0.0451 0.4085 A12 0.0033 0.0445 0.4032 B11 0.0033 0.0445 0.4032 B12 0.0033 0.0445 0.4032 A21 0.0026 0.0352 0.3192 A23 0.0026 0.0349 0.3163 B21 0.0023 0.0308 0.2785 B22 0.0023 0.0308 0.2785 B33 0.0019 0.0261 0.2366 A34 0.0019 0.0261 0.2366 A31 0.0019 0.0261 0.2366 C21 0.0019 0.0260 0.2357 C33 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 A41 <td>A22</td> <td>0.0052</td> <td>0.0705</td> <td>0.6383</td>	A22	0.0052	0.0705	0.6383
C22 0.0039 0.0520 0.4713 A11 0.0033 0.0451 0.4085 A12 0.0033 0.0445 0.4085 B11 0.0033 0.0445 0.4032 B12 0.0033 0.0445 0.4032 A21 0.0026 0.0352 0.3192 A23 0.0026 0.0349 0.3163 B21 0.0023 0.0308 0.2785 B22 0.0023 0.0308 0.2785 B33 0.0019 0.0261 0.2366 A34 0.0019 0.0261 0.2366 A31 0.0019 0.0261 0.2366 C21 0.0019 0.0260 0.2357 C31 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 A41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 <td>B31</td> <td>0.0041</td> <td>0.0552</td> <td>0.5000</td>	B31	0.0041	0.0552	0.5000
A11 0.0033 0.0451 0.4085 A12 0.0033 0.0451 0.4085 B11 0.0033 0.0445 0.4032 B12 0.0033 0.0445 0.4032 A21 0.0026 0.0352 0.3192 A23 0.0026 0.0349 0.3163 B21 0.0023 0.0308 0.2785 B22 0.0023 0.0308 0.2785 B32 0.0023 0.0308 0.2785 A33 0.0019 0.0261 0.2366 A34 0.0019 0.0261 0.2366 A31 0.0019 0.0260 0.2357 C31 0.0016 0.0211 0.1914 C33 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 A41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 <td>C23</td> <td>0.0039</td> <td>0.0520</td> <td>0.4713</td>	C23	0.0039	0.0520	0.4713
A12 0.0033 0.0451 0.4085 B11 0.0033 0.0445 0.4032 B12 0.0033 0.0445 0.4032 A21 0.0026 0.0352 0.3192 A23 0.0026 0.0352 0.3192 C11 0.0026 0.0349 0.3163 B21 0.0023 0.0308 0.2785 B22 0.0023 0.0308 0.2785 A33 0.0019 0.0261 0.2366 A34 0.0019 0.0261 0.2366 A31 0.0019 0.0260 0.2357 C31 0.0016 0.0211 0.1914 C33 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 C41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 0.0011 0.0154 0.1393 A13 0.0011 0.0152 0.1379 A15 <td>C22</td> <td>0.0039</td> <td>0.0520</td> <td>0.4713</td>	C22	0.0039	0.0520	0.4713
B11 0.0033 0.0445 0.4032 B12 0.0033 0.0445 0.4032 A21 0.0026 0.0352 0.3192 A23 0.0026 0.0349 0.3163 B21 0.0023 0.0308 0.2785 B22 0.0023 0.0308 0.2785 A33 0.0019 0.0261 0.2366 A34 0.0019 0.0261 0.2366 A31 0.0019 0.0261 0.2366 C21 0.0019 0.0260 0.2357 C31 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 A41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 0.0011 0.0154 0.1393 A13 0.0011 0.0152 0.1379 A15 0.0011 0.0152 0.1379 A15 <td>A11</td> <td>0.0033</td> <td>0.0451</td> <td>0.4085</td>	A11	0.0033	0.0451	0.4085
B12 0.0033 0.0445 0.4032 A21 0.0026 0.0352 0.3192 A23 0.0026 0.0352 0.3192 C11 0.0026 0.0349 0.3163 B21 0.0023 0.0308 0.2785 B22 0.0023 0.0308 0.2785 A33 0.0019 0.0261 0.2366 A34 0.0019 0.0261 0.2366 A31 0.0019 0.0261 0.2366 C21 0.0019 0.0260 0.2357 C31 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 A41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 0.0011 0.0154 0.1393 A13 0.0011 0.0152 0.1379 A24 0.0009 0.0117 0.1064 C12 <td>A12</td> <td>0.0033</td> <td>0.0451</td> <td>0.4085</td>	A12	0.0033	0.0451	0.4085
A21 0.0026 0.0352 0.3192 A23 0.0026 0.0352 0.3192 C11 0.0026 0.0349 0.3163 B21 0.0023 0.0308 0.2785 B22 0.0023 0.0308 0.2785 A33 0.0019 0.0261 0.2366 A34 0.0019 0.0261 0.2366 A31 0.0019 0.0260 0.2357 C31 0.0016 0.0211 0.1914 C33 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 A41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 0.0011 0.0154 0.1393 B24 0.0011 0.0152 0.1379 A15 0.0011 0.0152 0.1379 A24 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 A32 <td>B11</td> <td>0.0033</td> <td>0.0445</td> <td>0.4032</td>	B11	0.0033	0.0445	0.4032
A23 0.0026 0.0352 0.3192 C11 0.0026 0.0349 0.3163 B21 0.0023 0.0308 0.2785 B22 0.0023 0.0308 0.2785 A33 0.0019 0.0261 0.2366 A34 0.0019 0.0261 0.2366 A31 0.0019 0.0260 0.2357 C31 0.0016 0.0211 0.1914 C33 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 A41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 0.0011 0.0154 0.1393 B24 0.0011 0.0152 0.1379 A15 0.0011 0.0152 0.1379 A24 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 <td>B12</td> <td>0.0033</td> <td>0.0445</td> <td>0.4032</td>	B12	0.0033	0.0445	0.4032
C11 0.0026 0.0349 0.3163 B21 0.0023 0.0308 0.2785 B22 0.0023 0.0308 0.2785 A33 0.0019 0.0261 0.2366 A34 0.0019 0.0261 0.2366 A31 0.0019 0.0260 0.2357 C31 0.0016 0.0211 0.1914 C33 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 A41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 0.0011 0.0154 0.1393 B24 0.0011 0.0152 0.1379 A15 0.0011 0.0152 0.1379 A24 0.0009 0.0117 0.1064 C12 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0087 0.0785 A42 <td>A21</td> <td>0.0026</td> <td>0.0352</td> <td>0.3192</td>	A21	0.0026	0.0352	0.3192
B21 0.0023 0.0308 0.2785 B22 0.0023 0.0308 0.2785 A33 0.0019 0.0261 0.2366 A34 0.0019 0.0261 0.2366 A31 0.0019 0.0260 0.2357 C31 0.0016 0.0211 0.1914 C33 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 A41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 0.0011 0.0154 0.1393 B24 0.0011 0.0152 0.1379 A15 0.0011 0.0152 0.1379 A24 0.0009 0.0117 0.1064 C12 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0087 0.0785 A42 0.0006 0.0080 0.0728 P7	A23	0.0026	0.0352	0.3192
B22 0.0023 0.0308 0.2785 A33 0.0019 0.0261 0.2366 A34 0.0019 0.0261 0.2366 A31 0.0019 0.0260 0.2357 C21 0.0019 0.0260 0.2357 C31 0.0016 0.0211 0.1914 C33 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 A41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 0.0011 0.0154 0.1393 A13 0.0011 0.0152 0.1379 A15 0.0011 0.0152 0.1379 A24 0.0009 0.0117 0.1064 C12 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0087 0.0785 A42 0.0006 0.0080 0.0728 P7	C11	0.0026	0.0349	0.3163
A33 0.0019 0.0261 0.2366 A34 0.0019 0.0261 0.2366 A31 0.0019 0.0261 0.2366 C21 0.0019 0.0260 0.2357 C31 0.0016 0.0211 0.1914 C33 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 A41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 0.0011 0.0154 0.1393 A13 0.0011 0.0152 0.1379 A15 0.0011 0.0152 0.1379 A24 0.0009 0.0117 0.1064 C12 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0087 0.0785 A42 0.0006 0.0080 0.0728 P7	B21	0.0023	0.0308	0.2785
A34 0.0019 0.0261 0.2366 A31 0.0019 0.0261 0.2366 C21 0.0019 0.0260 0.2357 C31 0.0016 0.0211 0.1914 C33 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 A41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 0.0011 0.0154 0.1393 A13 0.0011 0.0152 0.1379 A15 0.0011 0.0152 0.1379 A24 0.0009 0.0117 0.1064 C12 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0087 0.0785 A42 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6	B22	0.0023	0.0308	0.2785
A31 0.0019 0.0261 0.2366 C21 0.0019 0.0260 0.2357 C31 0.0016 0.0211 0.1914 C33 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 A41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 0.0011 0.0152 0.1379 A13 0.0011 0.0152 0.1379 A24 0.0009 0.0117 0.1064 C12 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0087 0.0785 A42 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6	A33	0.0019	0.0261	0.2366
C21 0.0019 0.0260 0.2357 C31 0.0016 0.0211 0.1914 C33 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 A41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 0.0011 0.0152 0.1379 A13 0.0011 0.0152 0.1379 A15 0.0011 0.0152 0.1379 A24 0.0009 0.0117 0.1064 C12 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0087 0.0785 A42 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8	A34	0.0019	0.0261	0.2366
C31 0.0016 0.0211 0.1914 C33 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 A41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 0.0011 0.0152 0.1379 A13 0.0011 0.0152 0.1379 A24 0.0009 0.0117 0.1064 C12 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0087 0.0785 A42 0.0006 0.0080 0.0728 A43 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9	A31	0.0019	0.0261	0.2366
C33 0.0016 0.0211 0.1914 C32 0.0016 0.0211 0.1914 A41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 0.0011 0.0152 0.1379 A13 0.0011 0.0152 0.1379 A15 0.0011 0.0152 0.1379 A24 0.0009 0.0117 0.1064 C12 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0087 0.0785 A42 0.0006 0.0080 0.0728 A43 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9	C21	0.0019	0.0260	0.2357
C32 0.0016 0.0211 0.1914 A41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 0.0011 0.0152 0.1379 A13 0.0011 0.0152 0.1379 A15 0.0011 0.0152 0.1379 A24 0.0009 0.0117 0.1064 C12 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0087 0.0785 A42 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10	C31	0.0016	0.0211	0.1914
A41 0.0012 0.0161 0.1456 B23 0.0011 0.0154 0.1393 B24 0.0011 0.0154 0.1393 A13 0.0011 0.0152 0.1379 A15 0.0011 0.0152 0.1379 A24 0.0009 0.0117 0.1064 C12 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0087 0.0785 A42 0.0006 0.0080 0.0728 A43 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10	C33	0.0016	0.0211	0.1914
B23 0.0011 0.0154 0.1393 B24 0.0011 0.0154 0.1393 A13 0.0011 0.0152 0.1379 A15 0.0011 0.0152 0.1379 A24 0.0009 0.0117 0.1064 C12 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0087 0.0785 A42 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4	C32	0.0016	0.0211	0.1914
B24 0.0011 0.0154 0.1393 A13 0.0011 0.0152 0.1379 A15 0.0011 0.0152 0.1379 A24 0.0009 0.0117 0.1064 C12 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0087 0.0785 A42 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	A41	0.0012	0.0161	0.1456
A13 0.0011 0.0152 0.1379 A15 0.0011 0.0152 0.1379 A24 0.0009 0.0117 0.1064 C12 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0080 0.0728 A42 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	B23	0.0011	0.0154	0.1393
A15 0.0011 0.0152 0.1379 A24 0.0009 0.0117 0.1064 C12 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0080 0.0728 A42 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	B24	0.0011	0.0154	0.1393
A24 0.0009 0.0117 0.1064 C12 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0080 0.0728 A42 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	A13	0.0011	0.0152	0.1379
C12 0.0009 0.0116 0.1054 C13 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0080 0.0728 A42 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	A15	0.0011	0.0152	0.1379
C13 0.0009 0.0116 0.1054 A32 0.0006 0.0087 0.0789 A14 0.0006 0.0087 0.0785 A42 0.0006 0.0080 0.0728 A43 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	A24	0.0009	0.0117	0.1064
A32 0.0006 0.0087 0.0789 A14 0.0006 0.0087 0.0785 A42 0.0006 0.0080 0.0728 A43 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	C12	0.0009	0.0116	0.1054
A14 0.0006 0.0087 0.0785 A42 0.0006 0.0080 0.0728 A43 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	C13	0.0009	0.0116	0.1054
A42 0.0006 0.0080 0.0728 A43 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	A32	0.0006	0.0087	0.0789
A43 0.0006 0.0080 0.0728 P7 0.0064 0.1294 1.0000 P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	A14	0.0006	0.0087	0.0785
P7 0.0064 0.1294 1.0000 P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	A42	0.0006		0.0728
P6 0.0062 0.1257 0.9718 P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	A43	0.0006	0.0080	0.0728
P5 0.0058 0.1182 0.9137 P8 0.0056 0.1133 0.8755 P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	P7	0.0064	0.1294	1.0000
P8 0.0056 0.1133 0.8755 P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	P6	0.0062	0.1257	0.9718
P9 0.0050 0.1013 0.7827 P1 0.0046 0.0938 0.7253 P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	P5	0.0058	0.1182	0.9137
P1 0.0046 0.0938 0.7253 P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	P8	0.0056	0.1133	0.8755
P10 0.0045 0.0916 0.7080 P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	P9	0.0050	0.1013	0.7827
P2 0.0040 0.0799 0.6178 P4 0.0038 0.0772 0.5969	P1	0.0046	0.0938	0.7253
P4 0.0038 0.0772 0.5969			0.0916	0.7080
			0.0799	
P3 0.0034 0.0696 0.5383		0.0038	0.0772	
	P3	0.0034	0.0696	0.5383

6. Discussion

Valuable insights could be gained from this comprehensive evaluation of SM programs using ANP. In cluster-1, economic growth (B) is preferred over social well-being (C) which ranks second and environmental stewardship (A) which ranks third. Economic and social dimensions have almost equal priority weight, which means that manufacturing firms must focus on economic gains and the resulting social impact (stakeholders' welfare including those of employees, customers and community) equally, than on the decisions made for maximizing these gains separately. Addressing social concerns as a result of economic decisions could be attained via environmental impact on manufactured products and manufacturing processes. This claim is supported by the ranking of cluster-2 elements. Customer (C2), investment (B3), cost (B2), emissions (A2) and pollution (A1) are top-priority elements. Refining the details of this ranking can be done by taking a look at the top-priority elements in cluster-3. Customer satisfaction (C22), inclusion of customer rights (C23), investment in research and development (B31), community development (B32), revenue (B11), profit (B12), toxic substance (A11), GHG emissions (A12) and air emissions (A22) are top priority in cluster-3. Thus, decision-making in manufacturing must focus on maximizing revenue and profit by strategizing investment on research and development in technology and investment to contribute community development. The way to community development is to develop programs that minimize environmental impact of toxic substance, GHG emissions and air emissions. Revenue and profit are maximized by strengthened customer satisfaction and inclusion of customer rights to manufactured products. Developing programs that simultaneously enhance customer satisfaction and community development by addressing environmental concerns on toxic substance, GHG emissions and air emissions is fundamental to the increase of revenue and profit. Long-term strategy must address customer and community through environmental concerns so that sustainability is attained. This ranking influences the priority ranking of SM programs. The rank is as follows: elimination of PVC in plastic packaging (I7), adoption of "green" molding compound (I6), elimination of lead in the plating process (I5), energy efficient products (I8) and lean six sigma programs. The first three programs, which are cleaner production technologies, are directed at satisfying customer requirements while enhancing community development. Cleaner production on a wider scale can contribute to the greater welfare of society, as a society is the direct stakeholder in environmental concerns, arising from manufacturing processes (Singh et al., 2007). The last two programs focus on increasing profit by enhancing product research and development.

7. Conclusions and future work

This paper presents a comprehensive evaluation of SM programs using ANP. The comprehensiveness of such evaluation lies in the use of a recently concluded study of the US National Institute of Standards and Technology (US NIST) concerning the set of sustainability indicators derived from established and well-known indicator sets. Due to the emergence of a multi-criteria evaluation as a result of this use and due to the complexity of decision components in the evaluation, analytic network process (ANP) is used. ANP is deemed appropriate not only because of the multi-criteria nature of the evaluation process but primarily because of the presence of subjective components that are interrelating in complex relationships. An empirical study is carried out in a semiconductor manufacturing firm in the Philippines in order to evaluate the existing programs toward sustainability using the proposed evaluation framework. The results show that cleaner production technologies, i.e. elimination of PVC in plastic packaging, adoption of green molding compound and elimination of lead in the plating process, are considered top priority programs. Developing energy efficient products and adopting lean six sigma programs are considered second on the list. This paper suggests that sustainability is achieved by formulating strategies that enhance customer and community well-being via addressing environmental concerns especially on toxic substance, GHG emissions and air emissions.

Certain limitations are recognized in this study which are potential challenges for future work. This paper assumes that judgment elicitation is represented by crisp values. Future research could be extended by using fuzzy set theory to address vagueness in decision-making. An industry-wide evaluation could be done using the proposed framework to obtain more general insights regarding appropriate SM programs. Since preferences in evaluation may change over time due to technological, economic and political factors, dynamic judgment could be carried out to explore relevant or hybrid programs which are appropriate at different times in the planning horizon.

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TODIM-FSE: A MULTICRITERIA CLASSIFICATION METHOD BASED ON PROSPECT THEORY

Abstract

This paper introduces TODIM-FSE, a multicriteria method for classifying alternatives based on Prospect Theory. TODIM-FSE therefore relies on the TODIM method combined with the Fuzzy Synthetic Evaluation approach. TODIM-FSE makes use of the innovative "contribution" concept, not used previously for multicriteria classification purposes. This notion is central to the classification procedure of TODIM-FSE as it is associated to the contribution of each criterion to the classification of a given alternative in a predefined category. The TODIM-FSE method is explained in this paper by means of an application example and its steps are outlined. The application example has to do with the selection of trainee candidates for a company in the area of information technology. The classification of the candidates allows to identify the best of them, which is typically done at the first stage of the selection process. Some of the evaluation criteria considered in the study were: computers skills, mastery of technical English, and previous working experience in the field. In the second stage of that process another procedure ranks the best candidates. TODIM-FSE can be easily programmed in spreadsheets so as to be made available to professionals without a sound knowledge of either Multiple Criteria Decision Analysis or Prospect Theory. Currently the authors are working on a series of applications for validating TODIM-FSE in a broader way.

Keywords: TODIM-FSE method, Prospect Theory, Multicriteria classification of candidates, Multiple Criteria Decision Analysis.

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1. Introduction

This paper presents a new method for multicriteria classification of alternatives inspired by the TODIM method (Gomes and Lima, 1991, 1992; Gomes et al., 2009; Gomes and Rangel, 2009; Rangel et al., 2011; Moshkovitch et al., 2011; Gomes and González, 2012; Gomes et al., 2013). The TODIM-FSE method is also based on Prospect Theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992) and Fuzzy Synthetic Evaluation or FSE (Lu et al., 1999; Onkal-Engin and Demir, 2004; Chang et al., 2001; Sadiq et al., 2004; Kuo and Chen, 2006). While the TODIM method is a multicriteria method for ranking alternatives well established in the scientific literature, FSE, although not known as a multicriteria method has already been used as such (Kuo and Chen, 2006).

This paper intends to merge important features of both methods, TODIM and FSE to present an innovative multicriteria classification procedure. The classification is based on the "contribution" concept not used previously in MCDA, and along with other characteristics constitutes the body of the method. The role of Prospect Theory in TODIM-FSE is represented by the aggregation functions adapted in this paper to classify the alternatives.

The operation of the TODIM-FSE method is shown in this paper through a case study in human resources evaluation. The purpose of this evaluation is to select trainees for an information technology company. The company is highly rated in the labor market and offers attractive job opportunities for new professionals. Because of high demand, the process was divided in two stages. In the first stage the candidates are screened and the best ones identified. In the second stage the candidates selected in the first stage are evaluated in greater detail and rigor. This paper approaches the first stage of the process, where the candidates answer questions and take computerized tests. This information is used to classify them into four categories: excellent, very good, good or bad. Three evaluation criteria are used: computer skills, English language skills and working experience.

There exist few multicriteria methods to classify discrete alternatives. The book by Doumpos and Zopounidis (2002) gives detailed information on methods and techniques of multicriteria classification available in literature. The most widely known methods according to Zopounidis and Doumpos (2002) are ELECTRE TRI and UTADIS. Thus, TODIM-FSE is an option for typical applications for alternative classification using multiple sorting criteria.

This paper is divided in the following way: Section 2 gives a brief description of Prospect Theory taking into account the relevant aspects for the understanding of TODIM-FSE. Section 3 describes all the stages of the method. In section 4 these stages are used in a case study. Finally, the conclusions are presented.

2. Prospect Theory

Prospect Theory belongs to the field of cognitive psychology and describes how people make decisions under conditions of risk. Through a set of experiments performed in the 1970s Daniel Kahneman and Amos Tversky discovered previously unknown behavior. They observed that in situations involving gains people tend to be more conservative as regards risk, while in situations involving losses they are more prone to risk. Therefore, when people have a chance of winning, they prefer a lower but certain gain, than to risk for higher although uncertain gains. When a situation involves losses, people prefer to risk losing more but with the possibility of losing nothing than to suffer a smaller but certain loss. Additionally, the researchers noticed that situations involving losses are usually more relevant and striking than situations involving gains. This behavior is graphically represented in their seminal paper (Kahneman and Tversky, 1979) by a value function which is extremely relevant to understand the equations used in the TODIM-FSE method. Figure 1 illustrates this behavior. From the use of this value function within a multicriteria context, people's satisfaction can be quantitatively measured by entering into the model the characteristics of risk aversion and risk seeking, natural to people.

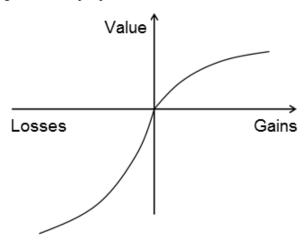


Figure 1. Value function of Prospect Theory

Although when ranking alternatives in the presence of multiple criteria we are not necessarily dealing with lotteries, the idea of being risk-averse in the domain of gains and risk-prone in the domain of losses is subject to the mathematical description by the value function of TODIM. This value function is built step-by-step as it will be shown in section 3.6. A detailed explanation of the TODIM method can be found, for example, in Gomes et al. (2009; 2013).

3. The TODIM-FSE method

As previously mentioned, TODIM-FSE is a method for multicriteria classification of discrete alternatives inspired by the TODIM method and by the Fuzzy Synthetic Evaluation (FSE).

In order to facilitate the understanding and use of the method, TODIM-FSE is described here step-by-step following the example of Goodwin and Wright (2004) when they described the SMART method (Edwards, 1977). However, the steps below do not need to be strictly followed in the sequence proposed.

- Step 1: Determining decision makers and decision analysts.
- Step 2: Analyzing and structuring the decision making problem.
- Step 3: Defining the relevant criteria of the problem.
- Step 4: Defining categories and contribution functions.
- Step 5: Defining the relative weights of the criteria.
- Step 6: Classifying each alternative to one of the categories.
- Step 7: Validation Analysis.

Each stage is described in detail below.

3.1. Step 1: Determining decision makers and decision analysts

This stage is used to determine the persons involved in the decision making process. Decision makers are the individuals who actually make decisions regarding the problem. They define the criteria to be used and their judgments (criteria weights, evaluation of the alternatives according to the criteria, etc.) contribute to construct the final result. The decision analysts are individuals who know the decision aiding methods and therefore support the development of the decision making process.

3.2. Step 2: Analyzing and structuring the decision making problem

It is very important to analyze the problem and discuss it thoroughly, to be certain that the right problem is being addressed. Ill-defined problems often lead to good decisions for the wrong problem. In this way, all the effort undertaken becomes useless. References on the subject can be found in Belton and Stewart (2010).

3.3. Step 3: Defining the relevant criteria of the problem

The construction of the decision making model begins in this step. The decision makers suggest the criteria to be considered for classifying the alternatives through brainstorming. The criteria are then screened, combined or eliminated to meet the recommendations of Keeney and Raiffa (1976) for the construction of

a good set of criteria. According to those two authors, the criteria set must present the following characteristics: operationality, decomposability, minimum size, completeness and non-redundancy.

3.4. Step 4: Defining categories and contribution functions

Once the criteria are established, the next step is defining the number of categories (denoted below by "cat") to be used in the model. As a rule of thumb, no more than five categories should be used. In this manner, the model becomes simpler, more attractive and easy to use. Once the number k of categories is defined, the contribution values (represented by μ) that each criterion provides to classify an alternative within a certain category must also be defined. The concept of contribution in the sense used in TODIM-FSE is, to the best of our knowledge, innovative.

Contribution values should vary continuously between 0 (zero) and 1 (one), with the value 1 (one) indicating that the criterion has the greatest contribution to the classification of an alternative within a given category. The value 0 (zero) indicates that the criterion does not contribute to the classification of an alternative within a given category. Intermediate contribution values are also allowed. It is important to note the similarity to the concept of values of membership functions in fuzzy set theory (Mendel, 1995; Zadeh, 2008). The contribution values are defined in a different way for qualitative and quantitative criteria. If the criterion is qualitative, we expect that its evaluation γ is done on a scale with discrete values. The contribution values for each category are defined for each verbal value γ of the scale, in the form of contribution tables, as shown in Table 1. A set of contributions, represented by the corresponding row in the table, is defined for each possible evaluation γ assigned to criterion i.

Table 1 Contributions table for qualitative criterion *i*

		Categories					
Evaluation	Cat 1	Cat 2		Cat k-1	Cat k		
γ1	μ_{11}	μ_{12}	•••	μ_{1k-1}	μ_{1k}		
γ ₂	μ_{21}	μ_{22}	•••	μ_{2k-1}	μ_{2k}		
			•••		•••		
γ _m	μ_{m1}	μ_{m2}		μ_{mk-1}	μ_{mk}		

A quantitative criterion can take continuous values. In this case contributions are represented by contribution functions, which are similar in shape and construction to membership functions in fuzzy set theory. However, one important

point to highlight is that, despite the fuzzy set similarity, that is an unnecessary knowledge to the method's users once they don't need to know what are these sets to build the contribution functions. Figure 2 illustrates an example of contribution functions described with sigmoid functions for the three categories. For the value 42 of the criterion the following contribution vector associated to each category is obtained: [0.19 0.78 0]. Thus, when the criterion j takes the value 42 it is contributing with 0.19 to the classification of the alternative in the first category, 0.78 to the classification of the alternative in the second category and with 0 to the classification of the alternative in the third category.

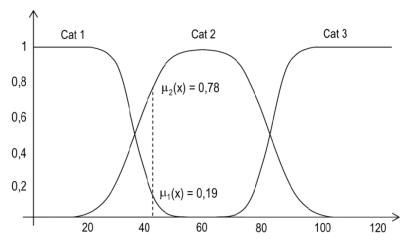


Figure 2. Contribution functions for quantitative criterion *j* in a problem with three categories

Once the contribution tables or contribution functions for each criterion are defined, it is possible to group the first data set relevant to the model, here called the *table of criteria grouped contributions*. Each row of this table is obtained from the evaluation of an alternative using each criterion. For the qualitative criteria they represent a row in Table 1. For the quantitative criteria they represent the value of the contribution function associated with the quantitative value assigned to the criterion *j*. Table 2 shows an example of such a table. Note that each row of this table is formed by the contributions associated with the evaluation made for each criterion. For a qualitative criterion the row associated with the evaluation is extracted from its contributions table. The extracted row is copied in the corresponding row of Table 2. For a quantitative criterion, the contribution vector obtained is copied on the corresponding row of Table 2.

		Č 1						
		Categories						
Criterion	Cat 1	Cat 2		Cat _{k-1}	Cat k			
crit ₁	μ_{11}	μ_{12}		μ_{1k-1}	μ_{1k}			
crit ₂	μ_{21}	μ_{22}		μ_{2k-1}	μ_{2k}			
		•••		•••				
crit _n	μ_{n1}	μ_{n2}		μ_{nk-1}	μ_{nk}			

Table 2
Table of criteria grouped contributions

One important point is that the decision maker must evaluate each alternative according to each criterion, by determining the contribution values for each category. This information must be generated and will serve as an input to the rank procedure that will fit the alternative (and the classification itself) in the best category, described later in step 6. For this reason, in the table of criteria grouped contributions, the same number of categories for different criteria is assumed.

3.5. Step 5: Defining the relative weights of the criteria

The second and last data set relevant for the model is defined in this step: the weights of criteria. Those weights are interpreted as measures of relative importance of criteria and must add up to 1.0. Therefore, the simplest way to obtain those weights is by direct assignment of values on a preset scale, followed by normalization. The result of both procedures is a weight vector W shown in (1) and (2).

$$W = [w_1 w_2 \dots w_{n-1} w_n] \text{ and }$$
 (1)

$$\sum_{i=1}^{n} w_i = 1 \tag{2}$$

The weights of criteria in TODIM or in its extension TODIM-FSE are measures of relative importance of criteria. By criteria importance we understand the power of the criteria in discriminating the overall desirability of the alternatives, as explained by Choo et al. (1999). In other words, the relative importance of a given criterion is a measure of the extent to which the rankings of the alternatives under that particular criterion are the same as their overall ranking.

3.6. Step 6: Classifying each alternative to one of the categories

For this step the two data sets relevant for the classification are already defined: the table of the criteria grouped contributions (Table 2) and the weights of the criteria (1). Once we know the contribution of each criterion to the classification

of an alternative in a given category, we use the trade-offs between the criteria weights and aggregate everything to find the category in which the alternative has the highest score (i.e., the class in which each alternative fits). This is done using the TODIM method. At this point n matrices of partial dominance Φ_c are being constructed, one for each criterion c. The elements of each matrix are given by (3):

$$\Phi_{c}(\text{cat}_{i,}\text{cat}_{j}) = \begin{cases}
\sqrt{\frac{w_{rc}(\mu_{ic} - \mu_{jc})}{\sum_{c=1}^{n} w_{rc}}}, \mu_{ic} - \mu_{jc} > 0 \\
0, \mu_{ic} - \mu_{jc} = 0 \\
-\frac{1}{\theta} \sqrt{\frac{(\sum_{c=1}^{n} w_{rc})(\mu_{jc} - \mu_{ic})}{w_{rc}}}, \mu_{ic} - \mu_{jc} < 0
\end{cases} (3)$$

In (3) we have:

 $\Phi_{c}(cat_{i}, cat_{j})$ - measure of dominance of category i (cat_i) over category j (cat_j) with respect to the criterion c;

 w_{rc} - tradeoff between a pre-chosen criterion r (denoted here as reference criterion) and the criterion c;

 $\mu_{ic} - \mu_{jc}$ – difference between the contributions to the classification of the *i*-th and the *j*-th evaluations in the criterion c (extracted from Table 2);

 $\sum_{c=1}^{n}w_{rc}$ – sum of the tradeoffs over all criteria;

 θ – a loss aversion parameter (i.e., attenuation factor of the losses);

 $\mu_{ic} - \mu_{ic} > 0$ measure of the gain, if this value is positive;

 $\mu_{ic} - \mu_{jc} = 0$ no gain and no loss reference point;

 $\mu_{\rm ic} - \mu_{\rm jc} < 0$ measure of the loss, if this value is positive.

The matrix Φ_1 , for instance, is constructed using only the contribution values associated with the criterion 1, that is, only the first row of the table of the criterion grouped contributions. The differences $\mu_{i1} - \mu_{j1}$ are seen as gains or losses associated with the value function of Prospect Theory, as represented graphically in Figure 1. If the difference is positive (indicating a dominance gain of the category i over the category j, in this case in the criterion 1) the value of the generic element a_{ij} of the matrix Φ_1 is given by the first segment of (3); if the difference is negative (indicating a dominance loss of the contribution of the category i over the category j) the value of the same element a_{ij} is given by the second segment of (3); and it is 0 if the difference is 0, corresponding to the second segment of (3). The values w_{rc} represent the weight of the criterion c divided by the weight of a reference criterion r (i.e. $w_{rc} = w_c/w_r$). In this case, the latter can be for example the criterion with the highest weight. It is easy to verify that it makes no difference which one is the reference criterion. The value θ is the at-

tenuation factor of the losses. Different choices of this value lead to different forms of the value function of Prospect Theory in the negative quadrant (Figure 1). Note, therefore, that the matrix Φ_c displays a set of dominance values of the categories with respect to each criterion.

Once the matrices of partial dominance for each criterion are calculated, the matrix of dominance $\delta(cat_i, cat_i)$ is calculated as shown in (4):

$$\delta(\operatorname{cat}_{i,}\operatorname{cat}_{j}) = \sum_{c=1}^{n} \Phi_{c}(\operatorname{cat}_{i,}\operatorname{cat}_{j}) \qquad \forall (i,j)$$
 (4)

Each element of the dominance matrix $\delta(\text{cat}_i, \text{cat}_j)$ sums all the partial dominances obtained previously from each criterion. The final result is obtained by calculating the vector Ξ with the general element ξ_i shown in (5):

$$\xi_{i} = \frac{\sum_{j=1}^{k} \delta(\text{cat}_{i,} \text{cat}_{j}) - \min \sum_{j=1}^{k} \delta(\text{cat}_{i,} \text{cat}_{j})}{\max \sum_{j=1}^{k} \delta(\text{cat}_{i,} \text{cat}_{j}) - \min \sum_{j=1}^{k} \delta(\text{cat}_{i,} \text{cat}_{j})}$$
(5)

The term $\sum_{j=1}^k \delta(\text{cat}_{i,}\text{cat}_{j})$ represents the sum of the elements from the i-th row of the matrix δ , the term $\min \sum_{j=1}^k \delta(\text{cat}_{i,}\text{cat}_{j})$ represents the least of these sums, and the term $\max \sum_{j=1}^k \delta(\text{cat}_{i,}\text{cat}_{j})$ represents the greatest sum. For this reason, according to (5), the vector Ξ will always have a component with value 1 (one) representing the most appropriate category for the classification, as well as another with value 0 (zero), representing the least adequate category for the classification. Intermediate values are assigned to the remaining categories.

3.7. Step 7: Validation analysis

The validation analysis is important for creating a good model to support decision making. The alternatives previously classified in each of the proposed categories are used as reference to adjust the classification produced by TODIM-FSE. These adjustments can be made in the criterion weights or in the contribution tables or functions.

4. Application example: evaluation of trainees for an information technology company

The IT Company operates in the area of computational technology and looks for young people with computer skills, among other requirements, to be trainees of the company. The company therefore wants to perform an initial screening of the best candidates. This stage is to be entirely performed through the company's web site. Each registered candidate has to answer a questionnaire and take tests to have his knowledge in the relevant areas assessed. From the responses to the question-

naire and the test scores obtained by the candidate, it is possible to classify him according to the TODIM-FSE method. In this manner, the score obtained by the candidate classifies him in one of the pre-established categories. This way, the TODIM-FSE method produces the desired screening of all candidates.

4.1. Determining decision makers and decision analyst

Decision makers are the senior executives of the IT company responsible for the selection process and the decision analyst is the manager of that process.

4.2. Analyzing and structuring the decision making problem

To better understand the work to be undertaken it is important to present a summary of the practical use of the TODIM-FSE method, and the type of inputs to be supplied from the senior executives. Then, the senior executives explained their goals, the desired type of professional and the plans for those professionals. In this way, the problem was well understood by the decision makers and the decision analyst, ensuring reliability in the evaluation process.

4.3. Defining the relevant criteria of the problem

After discussing the desired profile of the new trainees it was possible to define the criteria to be taken into consideration in the selection process, which are: a) Computer knowledge, b) English language skills, c) Working experience d) Interpersonal relationship skills. These criteria are described in more detail below:

- a) Computer knowledge: the IT company needs candidates with an extensive knowledge in computer science, familiar with both office applications and programming languages. A multiple choice test will be used to evaluate the candidate's knowledge including several questions on computer topics deemed important by the company.
- b) *English language skills*: the company believes that it is very important that the candidate read and speak English. However, at this stage only reading and understanding skills will be evaluated. Again, a multiple choice test will be used.
- c) Working experience: working experience will be assessed by a questionnaire which tries to identify the quality of the candidate's working experience. The candidates should be young, and for this reason not much is expected in this respect. It will be used as a differential.
- d) *Interpersonal relationship skills*: this criterion is considered very important: personal relationships, teamwork, and verbal communication skills should be taken into consideration. However, as this criterion requires personal contact with the candidate, it will be left to the next stage of the evaluation.

Therefore only the first three criteria will be used at this stage of the evaluation: a) Computer knowledge, b) English language skills, c) Working experience. It is worth noting that these three criteria meet almost all the characteristics that Keeney and Raiffa (1976) suggest for a criterion set, particularly non-redundancy. They do not meet the completeness requirement because it is not possible to evaluate "Interpersonal relationship skills" at this stage.

4.4. Defining categories and contribution functions

Four (4) evaluation categories were defined for this problem: excellent, very good, good and bad. The contributions of each criterion are determined from these categories. Computer knowledge and English language skills are handled as quantitative criteria because they are scored based on the result of a test. As described previously, in this step it is necessary to build the range of contribution values (µ) for each category (represented by contribution functions), that allow to determine the contribution of each criteria evaluation to the classification of the alternative in each category. Thus, contribution functions using trapezoidal functions are defined for them as shown in Figure 3. Figures 3 and 4 were created using Microsoft Excel, which was also used to obtain the results of this example. It is possible to see (looking at Figure 3) that, according to the decision maker's judgments, for example, a candidate who obtains a grade between 0 (zero) and 5 (five) in both tests, may receive the greatest contribution value (1) to the "bad" category, and zero to other categories. If the candidate receives a grade greater that 6 (six) in both tests the contribution values for the "bad" category will be zero, and for grades between 5 (five) and 6 (six) we obtain contribution values within the interval (0,1). The same contribution functions are used for both criteria. Since we know the score obtained by the candidate in the multiple choice tests, the contribution of each criterion to the classification the candidate in a given category is obtained.

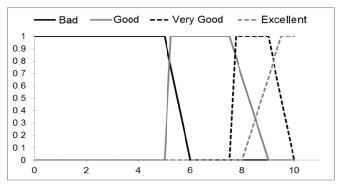


Figure 3. Contribution functions for Computer knowledge and English language skills criteria

The criterion *Working experience* is handled as a qualitative criterion and thus a contribution table, as shown in Table 3, can be defined. From the answers to this criterion questionnaire it is possible to assess whether the candidate has previous working experience and, in this case, whether the experience is related to the position to be filled.

Table 3 Contribution table for the *Working experience* criterion

	Categories			
Assessment	Bad	Good	Very good	Excellent
Worked in the computer science area	0	0.5	0.8	1
Worked in a technical area different from computer science	0.5	0.8	1	0.8
Worked in a non-technical area	0.8	1	0.8	0
Has no working experience	1	0	0	0

4.5. Defining the relative weights of the criteria

The relative weights of criteria are determined from direct assignment on a scale from 0 to 100. After normalization the following weights are obtained: $w_{Ck} = 0.605$; $w_{ME} = 0.283$; $w_{We} = 0.112$.

4.6. Classifying each alternative in one of the preset categories

To evaluate and classify candidates, it is necessary to obtain the scores and the answers of a given candidate, as previously explained. With this information it is possible to obtain input data for the classification using TODIM-FSE, as shown in Table 4.

Table 4
Results of the scores obtained by the candidate in both tests (*Computer knowledge* and *Mastering of the English language*) and the questionnaire on working experience

Criterion	Candidate evaluation
Computer knowledge	8.5
Mastering of the English language	9.0
Working experience	Has no experience

The *table of criteria grouped contributions* (represented by Table 5, for this particular candidate) is obtained from that input data. The first and the second rows were extracted from the contribution functions defined in Figure 3. The third row was obtained from the last row of Table 3. With Table 5 and the criterion weights it is possible to classify the candidate by using equations (3), (4) and (5).

Table 6

Table of criteria grouped contributions for a particular candidate

Criterion	Bad	Good	Very Good	Excellent
Computer knowledge	0.00	0.33	1.00	0.33
Mastering of the English language	0.00	0.00	1.00	0.67
Working experience	1.00	0.00	0.00	0.00

Table 6 shows the candidate's final classification. All the categories will receive a score. However, only the category with the highest score will be chosen.

Final classification of the "very good" candidate

Final Classification			
Bad	0.25		
Good	0.00		
Very Good	1.00		
Excellent	0.43		

4.7. Validation analysis

The validation analysis is then performed aiming at checking if the tests have indeed properly classified the candidates. Note that if the tests are too easy, even not well qualified candidates can obtain a good evaluation. Conversely, if the tests are too difficult, very good candidates may be incorrectly classified as "bad". For this reason, before placing the model in the automatic evaluation system, the test was applied to employees considered "very good" or "excellent" in the *Computer knowledge* and *English language skills* areas to allow for adapting the contribution functions to the test level. This means that if the tests are too difficult the contribution functions can be modified to classify candidates with lower score in higher categories. Conversely, if the tests are too easy, the contribution functions may be modified to classify only the candidates with very high score in the best categories.

It is important to stress that only the criteria with judgment values defined by the test were subject to this type of analysis. This is not really necessary for the *Working experience* criterion.

Two (2) experienced employees (here denoted as Employee 1 and Employee 2) who are generally considered excellent by the company are chosen to test the level of the *Computer knowledge* and *English language skills* tests. These employees took the tests without any prior preparation and obtained the scores shown in Table 7.

Table 7

Results of the scores obtained by the two employees in the *Computer knowledge* and *English language skills* tests

Tests	Employee #1	Employee # 2
Computer knowledge	8.0	7.5
English language	8.5	8.0

As these 2 employees were considered excellent both in *Computer knowledge* and *Mastering of the English language*, the scores that they obtained were considered low, indicating the high level of both tests. Thus, to properly classify the prospective candidates the contribution functions shown in Figure 3 were modified. Their modifications are shown in Figure 4.

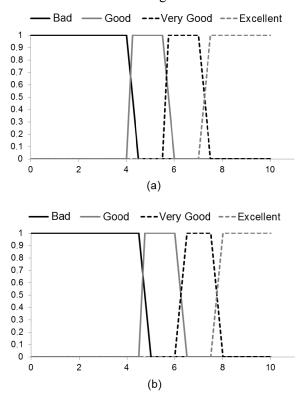


Figure 4. Contribution functions modified for the criteria (a) Computer knowledge and (b) English language skills

The modification brought a direct impact on the classification of the candidate evaluated. He was previously considered "very good" and after the modification he was considered "excellent". Table 8 shows the new scores of the candidate in each category.

Table 8
The new score of the candidate formerly considered "very good" ("excellent")
after the validation analysis

Final Classification		
Bad	0.56	
Good	0	
Very Good	0	
Excellent	1.00	

5. Conclusions

TODIM-FSE proved to be effective for classifying the prospective candidates in the study case. An important characteristic of the method is its simplified mathematical formulation, without pre-requirements as in the UTADIS classification method (Zopounidis and Doumpos, 2002), which uses linear programming in its formulation. This enables users with little training to use it without difficulty. The validation analysis, last step of the process is not required in the classification process; and it is not performed in widely used classification methods such as UTADIS and ELECTRE TRI (Doumpos and Zopounidis, 2002). However, it was very important in obtaining the final result because it corrected the candidate's classification. But the validation analysis may not be important for some criteria used in the classification process.

The main differentials of the method are: (1) use of the "contribution" concept indicating the contribution of a criterion to the classification of the alternative in a given category and (2) consideration of the Prospect Theory, embedded in the TODIM method equations used in TODIM-FSE. Strictly speaking, it would be possible to use another method for ranking categories, substituting the TODIM method in step 6. However, the last differential would be lost.

Although the contribution functions described in Figures 3 and 4 are similar to fuzzy sets, it is worth noting that this knowledge is not necessary to construct them.

The consolidation of the method still demands a large number of applications to test and improve TODIM-FSE.

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Vol. 9

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MULTICRITERIA EVALUATION OF PIPE ORGAN CONSTRUCTION PROJECTS

Abstract

The construction of a pipe organ is a heterogeneous problem. Each interior, for which the instrument is planned, has its individual architectural and acoustic characteristics. The design of the instrument must be matched to the interior and therefore it will be individual and usually unique. Each investor commissioning a pipe organ also has his/her individual taste, preferences, and budget. These most important factors make the construction of a pipe organ a sum of the various relationships and a result of the willingness to compromise between objective factors and preferences of people. This paper presents the issue as a multiobjective task, in which we consider various criteria, such as size, volume, palette of timbres, etc., and show how the various options are presented to the investor. Will the best designers' solution be accepted by the investor and his/her budget? We should handle the various criteria so as to satisfy the investor without compromising the quality of the instrument.

Keywords: pipe organ construction, pipe organ sound project, multicriteria decision problem.

1. Introduction

The organ is an instrument belonging to the group of keyboard aerophones, in which the sound is created by the vibrations of the air column in the pipe. Due to the presence of *reed pipes* in many instruments whose sound source is a metal reed made to vibrate by the compressed air, the organ is also classified as an idiophone.

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Organs were built already in antiquity. In Christian Europe they appear in the high Middle Ages. Their modern evolution, both technological and timbral, has been taking place since the tenth century. In each stylistic epoch (Middle Ages, Renaissance, Baroque, Classicism, Romanticism, revival styles, new styles in the 20th century, modern times) organs were characterized by different features, drawing on the styles then used, but they always had individual characteristics, given by the designer and bearing the mark of the creator's individuality.

Organ are built by organ builders having at their disposal an appropriate workshop equipped to process materials necessary for the construction of the instrument: various kinds of wood, metals, leather, textiles, etc.

The timbral variety of the organ is visible above all in its disposition. An organ stop is a set of pipes of homogeneous construction, characterized by a uniform timbre. An organ can have several stops, from a few to several tens or even hundreds, depending on the size of the entire instrument. Stops are grouped into timbral sections controlled by various keyboards (e.g., section I or II of the consecutive manual, or the pedal section).

The organ is a musical instrument which for several centuries of its history has been characterized, more than any other instrument, by a great richness of form and size, both as regards its appearance and its sound. This is related to the constantly changing taste of the society, that is, to the stylistic eras in which the organ builders lived and worked, as well as to the technological progress in the manufacturing of the individual components of the organ. Above all, however, the variety in organ building from antiquity through modern times stems from the fact that there is no ready-made model of the instrument's appearance or sound. The design of an instrument is always adjusted to the given interior and to the expectations of the people who are directly interested in the construction of the given instrument.

For the given investor, several designs of an organ can be prepared, which differ significantly but are all based on the invariable parameters determined by the characteristics of the interior in which they should be realized. They may be regarded as decision variants which are worth evaluating with respect to various criteria. The purpose of the present paper is to attempt to define and order such criteria.

The paper is organized as follows. In section 2 we present selected issues related to the designing of an organ. In section 3 we describe the elements of an instrument and the possibilities of their shaping. In section 4 we present factors influencing an organ design in progress, while section 5 deals with the issue of the evaluation of an organ design as a multicriteria decision problem. The last section is a summary.

2. From the history of organ design

Theoreticians of organ building of earlier centuries focused on technological and material issues occurring during the process of perfecting this instrument, which has lasted incessantly since the Middle Ages¹. From among many outstanding personages, let us recall a few names, representative for the consecutive centuries: Arnolt Schlick (1511), Constanzo Antegnati (1608), Dom Bédos de Celles (1766-1778), and Johann Gottlob Töpfer (1855).

The visual aspect of the modern organ, originating in the Gothic model of the organ casing, developed autonomously in various regions of Europe, in accordance with the stylistic tendencies changing over the centuries. An analogous dependence is visible, to simplify matters greatly, as regards the variety of organ timbre until the end of the Baroque era. The Classical era brought a stagnation in this field, although in southern Germany, Austria, and Silesia, organ building was still booming.

Due to the transformations occurring in music at the turn on the 19th century, the organ ceased to satisfy the requirements of composers and performers, and as a result they fell out of fashion. Creative organ builders of the nascent Romantic style in music struggled to maintain the position which their instruments had held up to that time, starting new trends in the technology and technique of organ construction. Above all, they reformed the principles of timbral aesthetic of this instrument, creating, as a result, the so-called symphonic organ. The credit for this goes mainly to two organ builders: Eberhard Friedrich Walcker (1794-1872) (Moosmann, Schäfer, 1994) in Germany and Aristide Cavaillé-Coll (1811-1899) (Eschbach, 2005) in France.

The new Romantic style in organ building assumed a specific manner of designing their disposition². As time went by, this manner became so obvious that the idea of the organ timbre in the entire Europe was shaped by almost uniform patterns. Interesting directives in this respect can be found in the *Guide for the organists* by Antoni Sapalski (1880), probably the first work of this type written in Polish, published at the author's expenses in Cracow in the second half of the 19th century:

"The relationship of the size to the number of stops can be presented approximately in the following way, for instance:

In this paper we don't deal with the undisputed achievements of antiquity in the field of organ building, nor with the treatises from those times.

This statement is a simplification and does not take into account the differences of approach to registration of symphonic organ in various European countries. One should mention at least two previously listed, very different schools: French and German. But the timbral ideals and the conception of gradual dynamic changes remain consistent for the entire Romantic Europe.

- 1. To every two 8-ft³ stops one should add one 4-ft stop.
- 2. To every three 8-ft stops one should add one 4-ft stop and one 2-ft stop.
- 3. To every four 8-ft stops one should add one 6-ft stop (5-1/3 fifth), two 4-ft stops, one 2-ft stop, and a triple 2-ft mixture.
- 4. To each 16-ft stop one should add four 8-ft stops, one 6-ft stop, two 4-ft stops, one 2-ft stop, and a triple mixture or cornet.

It is difficult to state a rule based on this small example, which, however, serves as a kind of basis for the relationship of stop sizes which should be taken into account in the disposition of a planned organ".

Nobody had to ask Sapalski what stops exactly he had in mind, because everybody interested in the matter had a very similar idea as regards the organ style: the underlying Romantic aesthetics was taken for granted by all.

That era, like all others, had its end: in literature, in painting, in music, as well as in organ building. The slogan of revival reached its apogee several times, for instance in neoclassical architecture, or, later, in Romanesque revival or Gothic revival architecture. As regards the organ timbre, the return to Baroque models occurred, in the most advanced centers, at the turn of the 20th century, with the creation of the movement called *Orgelbewegung*, inspired by Albert Schweitzer. The creators of the new style turned against the Romantic tendencies, common in the organ building of that time, and postulated a return to the Baroque tradition, in particular to the ideals of the organ builders from the times of Johann Sebastian Bach. The principles of the 17th-century art of organ building had not been yet thoroughly investigated at that time, and therefore organs inspired by the assumptions of the *Orgelbewegung* have stylistic features characteristic both for the late Romanticism (intonation) and for the Baroque era (disposition features).

Theoreticians of organ building in the first half of the 20th century (Ellerhorst, 1936; Supper, 1855) gave the following exact guidelines for the relationship of the interior size to the instrument size:

- 1. In small interiors: for every increase by 50 cubic meters, there should be one stop added in the disposition.
- 2. In medium-size interiors: for every increase by 75 cubic meters, there should be one stop added in the disposition.
- 3. In large interiors: for every increase by 100 cubic meters, there should be one stop added in the disposition.
- 4. In interiors with capacity of:

Hara "O faat" rafara ta

Here "8-feet" refers to the length of the first, longest pipe, which decides about the pitch of the given stop. The longest the pipe, the lowest the pitch. Pipes are measured in feet, which is the historical unit of length.

- a) up to 100 persons 4-6 stops,
- b) up to 200 persons 8-12 stops,
- c) up to 400 persons 17-22 stops,
- d) up to 600 persons 30-40 stops,
- e) up to 1500 persons -70-80 stops.

Both Winfred Ellerhorst and Walter Supper referred here to the previously mentioned neo-Baroque style.

Over time it turned out that the the tendencies in organ building in the 20th century and the first decade of the 21st century reflect the polystylism of art in other disciplines. Those tendencies include stylization and avant-garde. When designing an organ, we reach to exact historical models (copies) or are inspired by the individual styles (stylistically oriented modern instruments); we strive to achieve a universality of the organ by mixing styles. As a result, although one can play music from any era on such an instrument, none will sound truly authentic (universalism). Moreover, using modern techniques, we build gigantic organs equipped in several improvements, often of startling performance possibilities.

The preference in organ building for specific styles⁴ and the bold expression of aesthetic opinions by the persons commissioning and designing the flagship masterpieces of the organ-building art in the last half a century prove, on the one hand, a high level of their awareness and organological knowledge and, on the other hand, show the multitude and the variety of solutions which can be applied in the process of designing an instrument for the specific interior.

To sum up, we can say that we live in times when the idea about the timbre and appearance of the organ is not homogeneous, as it was the case in the past. Nowadays we have at our disposal knowledge about styles, organ building experience, access to choice materials and techniques, thanks to which we can realize bold and varied designs. To take advantage of these possibilities we have to make decisions in various aspects, search for compromises or argue the legitimacy of "hard" conceptions and original solutions.

3. Elements of the instruments and possibilities of their shaping

An organ consists of:

- 1. Casing.
- 2. Console or keydesk (with the keyboards and couplers).
- 3. Prospect or façade pipes (visible) and pipes inside the case (invisible).
- 4. Wind chests (cases with valves, where the air is distributed to the individual pipes).

⁴ By a style we understand references to a specific era and region.

- 5. Tracker-action (a mechanism linking valves with keyboards).
- 6. Wind system (bellows, calcant, blowers).

In organ building, the following are evaluated:

- 1. Visual aspects:
 - the shape of the casing,
 - architecture,
 - the definable style (or its lack),
 - exposition of the instrument,
 - the console.

2. Timbral issues:

- the disposition of the organ (the number or stops and their types),
- a definable timbral style (or its lack),
- loudness and the ability to carry the sound,
- the timbral palette and the possibilities of the dynamic shaping of the sound.

3. Technical issues:

- type of windchest used,
- type of tracker-action used,
- selection of air pressure and of the type of wind system,
- equipment supporting the use of the instrument.

4. Factors influencing the design of an organ

Based on these fundamental and necessarily simplified issues, we shall attempt to name and define the factors that influence the design of an organ. We distinguish here objective and subjective (human) factors; next, we will describe the main features of the organ, influenced by these factors.

Objective factors:

- 1. Characteristic of the interior:
 - a) acoustics.
 - b) architecture and style.
- 2. The amount of space destined for the organ and its parameters:
 - a) bearing capacity,
 - b) surface area and height of the room,
 - c) shape of the room as well as architectural and structural obstacles.

Subjective (human) factors:

- 1. Individual aspirations and tastes/preferences of the investor.
- 2. Financial resources of the investor for the construction of the organ⁵.

⁵ One should remember that the organ is the most expensive instrument, intended to be used by several generations.

- 3. The conception of the designer.
- 4. Professional knowledge and experience of the organ builder.
- 5. The equipment of the organ-builder's workshop.
- 6. Musical and technical preferences of future users.

Each of the factors mentioned previously can influence the shaping of the individual parameters, gathered previously into three groups of issues. To simplify, we can describe this influence as follows:

1. Interior characteristics:

- a) Acoustics depending on the acoustic predispositions of the room, the designer estimates the required size of the instrument and the best localization of the instrument, and assesses which bandwidths are carried best in the interior and which need reinforcing, e.g. by multiplying them in the planned disposition. Moreover, knowing the acoustic parameters of the room, the designer can refer in his or her sound design to an historical sound style (such as the north-German Baroque style or else the diametrically different French Romanticism style), whose features will harmonize with the acoustic properties of the interior.
- b) Architecture and style depending on the results of acoustic research and the most appropriate suggested location of the instrument, the designer, in agreement with the investor, decides as to the localization of the instrument and for adapting to or else contrasting with the architectural style and interior decor. Knowing the size of the disposition planned, the organ builder determines the necessary volume of the organ casing to which the architect has to adapt the external appearance of the instrument.
- 2. The amount of space planned for the organ, and its parameters:
 - a) Bearing capacity the mechanism, case, and the pipes of a medium-tolarge instrument usually weigh from a few tons to more than ten tons. The planned location of the instrument has to be adequately prepared. In some justified cases it is necessary to perform additional alteration work, simultaneously with the work on the construction of the organ, to reinforce the place. This, too, has an impact on the costs of the enterprise.
 - b) Surface and height of the room it may be impossible to build an instrument of the size appropriate for the acoustics of the room, because of inadequate space or height of the room to house the organ. It is then necessary to make a compromise. For instance, two registers may have to be reduced to a single register, whose sound characteristics will be capable of "replacing" them.
 - c) Shape of the room and obstacles when the church choir or the alcove or balcony in the concert hall has a regular shape, there are no difficulties with the organ construction. Very often, however, difficulties arise, caused

by load-bearing beams protruding from the floor or ceiling, centrally situated windows, steep vault arches or cornices or architectural details which can't be removed. When designing an organ, it is necessary to adjust the design to the room shape and to carefully omit the obstacles. Also, windows, external walls, and the heating system often cause later degradation of the instrument, because of the exposure to solar radiation and problems with thermal wall insulation. For that reason, when designing the layout of the instrument, one should preserve appropriate distances from these obstacles or recommend additional construction work.

- 3. Individual aspirations and tastes/preferences of the investor depend on the level of his/her knowledge of the organ; they influence substantially the conception of the designer as well as the actions of the performer (organ builder).
- 4. Financial resources of the investor for the construction of the organ budget shortages limit not only the investor's aspirations and the designer's conception (who often has to choose less expensive solutions, against his/her opinion), but they also influence the organ builder, who is encouraged to limit the costs, which may result in a lower quality of the final product.
- 5. The designer's conception it has to follow the expectations of the investor and the users, it also has to correspond to the characteristics and parameters of the interior. Much depends in this matter on the qualifications and experience of the designer and on his/her ingenuity and imagination. The designer has to take into account the planned way of using the instrument (for instance, concert solo performance, accompaniment to singing during the liturgy, ensemble performance, playing with an orchestra, teaching). The designer has also to indicate the preferred technological solutions and suggest the layout of the individual sections of the instrument in the context of the acoustic properties of the interior and the planned timbral effect.
- 6. Professional knowledge and experience of the organ-builder it a necessary condition for the understanding and proper realization of the designer's idea.
- Equipment of the organ-builder's workshop lack of specialized, often very expensive tools makes it impossible in many cases to realize ambitious designs.
- 8. Musical and technological preferences of future users. We take into account professional users, technologically and scientifically prepared to use the instrument, who also have vast knowledge of the stylistic variety in organ-building. Each user has his/her own artistic taste whose influence on the conception and the way of building the instrument is proportional to the authority of the future user with the investor and performer.

The organ console can also be designed in many ways. In this case, what is evaluated are the appearance and ease of use. To facilitate the use of the instrument, modern technologies are applied nowadays, for instance, electronic technologies

nologies, whose application naturally raises the price. The shape of the console and the electronic aids are therefore a criterion which depends strongly not only on the user's preferences, but also on the investor's affluence.

5. Evaluation of organ-building designs as a multicriteria decision problem

The set of criteria discussed here depends for the most part on the place where the organ will be constructed, although to some extent it refers to universal issues. The basis is determined here by objective factors which we cannot (or want not) change (that is, first of all, volume and acoustics of the building). Based on this, we can prepare for the investor several organ designs, which will differ significantly. They can be defined as decision variants, which can be evaluated as regards, for instance, architecture of the casing, timbral style, solutions of instrument construction or ease of use. The definitions given in the previous sections constitute the first attempt to describe this phenomenon in a universal way. The order of the criteria and, in general, the consideration of variants of the individual groups of criteria depend on the interests of the investor and the expert. Once the variants and the assessment criteria for a specific realization are created, one should discuss the issue of measurement scales to be used for the individual criteria so as to best render the intentions of the persons performing the evaluation. In our research, we do not include any examples, since their thorough presentation would require a detailed description of all previously mentioned issues, which would be outside the scope of this paper. In the future, however, conducting such a process (be it hypothetical or supported by actual design and construction) and describing its results, seems well-founded.

6. Summary

The construction of an organ is often a compromise solution taking into account the factors listed above. Objectively definable conditions, such as: room size, results of acoustic research, technical expert opinions, interior style, can be assessed by various experts/designers in various ways. We start the assessment with timbral issues. Assuming solid preparation and knowledge of organ building by the group of experts, we obtain several good, but most likely differing designs, reflecting the tastes and preferences of each expert. Following this, the next group of experts presents their preferences as regards the shape of the organ casing which can house instruments with the expected sound characteristics. These experts are usually specialists in organ mechanics. They too take into account the objective factors researched previously, together with an additional proposal outlining the timbral characteristics of the organ. At the top of this

pyramid stands the investor, who is able, to some extent, to cope with the proposals presented. His financial resources and willingness to finance interesting, but not always necessary, solutions suggested by several experts are confronted with his own individual and artistic taste, both as regards the sound and (usually to a larger extent) the appearance of the instrument. The investor seeks the opinion of trusted persons and of the person who will use the instrument most often — the parish organist, the orchestra musician usually playing the organ part, etc.

Assuming a thorough and solid organ background of the closest advisor or of the investor himself, one could dispense with the "pyramid" of experts described above and, without problems, commission the construction of the instrument conceived by the experts directly from the investor's favorite organ building firm. This, however, happens extremely rarely, and the closest advisor of the investor is often a moderately educated organist led by his own comfort-seeking nature and not by the organ's quality.

Uncrowned king of musical tools, the most expensive of all instruments, living up to 200 and more years, certainly deserves to be the object of a solid and thorough multi-criteria project supported by well-founded knowledge and multi-disciplinary research in many disciplines of art and technology.

A few years ago Małgorzata Trzaskalik-Wyrwa had the opportunity to present a similar issue from the field of historical organ conservation. That, however, dealt with an already existing historical substance, to which one had to adapt the most suitable conservation-related decisions. In the case of the construction of a new organ, the weight of the particular criteria changes. The designer and the organ builder create a new reality, a new musical tool. Therefore, more possibilities appear for theoretical discussions BEFORE the start of its realization. AFTER the organ had been constructed and a large amount of money had been spent, it will not be easy to bear the critique when one has neglected to work out the design. Here we see a wide range of opportunities of applying multi-criteria methods, which – although they probably will not automate the decision-making process – will encourage to define the criteria precisely and will influence the awareness of the group of people interested in the realization of the project, as regards the weight and values of the actions undertaken and their influences on the final shape of the newly created organ.

To end this discussion, we quote again Antoni Sapalski's *Guide...* This quotation shows that now as in the past, attaching very high importance to the lowest price criterion is not the right method in organ building, since it impacts its quality: "It seems to follow that the size of the instrument should depend strictly on the size of the church. This should not, however, be always the guiding principle, since this instrument, among all known instruments, requires the largest amount of work, and therefore also higher costs; hence, imposing too much restrictions

in this respect on the organ builder puts him in the situation in which he is either unable to apply himself to the actually necessary size or else it is not possible to require of him to construct the instrument and perform its artistic completion" (Sapalski, 1880).

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