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THE ANALYSIS OF NEGOTIATORS' PREFERENCE CONSISTENCY IN INDIFFERENCE-SURFACE BASED SCORING SYSTEM

Abstract

In this paper we present a new method for analyzing the consistency of preferences of negotiators in building their scoring systems of negotiation offers. The method we propose can be used when the preferences are defined as general examples of full packages with the accompanying utility score, as it is done in the NegoManage negotiation support system in the conjoint analysis approach. During the preference elicitation stage the negotiators identify the indifference surfaces (or indifference sets) to which they also assign sample alternatives and scores. The verification of such the consistency of this assignment is based on the concept of the Jaccard index, that allows for measuring the similarity between fuzzy sets. Since we obtain a characteristics of equivalence sets in the form of probability distributions, which are further treated as fuzzy set membership functions, we can use the distribution characteristics to compute the Jaccard index for every pair of equivalence sets elicited from the negotiator. If these indexes are too high, the corresponding indifference sets should be reconsidered or integrated.

Keywords

Negotiation support, negotiation offers' scoring system, preference elicitation, indifference sets, kernel density estimation.

Introduction

In the process of multiple criteria decision-making (MCDM) the decision makers have to cope with problems of comparing and evaluating very many (usually conflicting) criteria. Such decision-making processes involve, depending on the decision context of the problem, evaluation, prioritization or selection of alternatives. Among many MCDM methods the most popular are: simple additive weighting models (based on multiple attribute utility theory) – [Keeney and Raiffa 1993], AHP [Saaty 1980; Saaty and Alexander 1989], ELECTRE [Roy and Bouyssou 1993] and PROMETHE [Brans 1982]. The MAUT-based models constitute a scoring system allowing for ranking any alternative after the weights and marginal utilities have been elicited. This method fuses single attribute utilities with weights assigned to attributes and results in a final value of utility for the given alternative. The AHP method is based on pairwise comparisons of attributes and alternatives, and results in a ranking of the given alternatives. ELECTRE and PROMETHEE methods are based on an outranking concept and give as a result of analysis an ordering of the given alternatives. The literature review shows quite a lot of examples of using these methods in solving actual business-related decision making problems [see eds. Figuera et al. 2005; Omkarprasad and Sushil 2006; Behzadian et al. 2010]. In the negotiation context, however, it is a simple additive weighting (SAW) model that is most widely used for eliciting negotiators' preferences. All the most popular negotiation support systems (NSSs) such as Inspire [Kersten and Noronha 1999], Negoist [Schoop et al. 2003] and SmartSettle [Thiessen and Soberg 2003] accomplish their decision support function by using the simple additive scoring model (sometimes hybridized with the conjoint analysis approach) for evaluating the negotiation template and building the scoring systems of the negotiation offers used in the actual negotiation phase for evaluation and analysis of the sequence of offers and counteroffers proposed by the parties as the negotiation contact proposals. But the recent negotiation experiments show that NSS users very often misinterpret the SAW scores and find it difficult to assign them to the negotiation options and issues [see Wachowicz and Kersten 2009; Paradis et al. 2010]. Therefore new mechanisms and systems are being built that apply preference elicitation approaches other than the SAW-based ones, such as NegoManage [Brzostowski and Wachowicz 2009, 2010] which allows to determine the scoring systems of negotiation offers deriving from the examples of offers that the negotiator specifies in the prenegotiation phase. The NegoManage system supports the negotiator in all phases of the negotiation process allowing not only for the preference elicitation but also for an exchange of offers and messages, tracking the negotiation history, negotiation profile identification and counterpart evaluation and selection. The preference elicitation engine is a key element of the system. The whole preference elicitation mechanism is based on the concept of the equivalence set that may be specified by the negotiator as a set of alternatives indifferent in terms of preferences. The negotiator also evaluates this set verbally by assigning to it a linguistic value of utility. The whole process of preference analysis requires of the negotiator the specification of the sequence of indifference sets with the corresponding utilities that are the basis for the scoring system of negotiation offers. After the scoring system has been prepared any alternative from the set of feasible alternatives (i.e. those defined in the template) can be evaluated in terms of utility assigned to this alternative. Since the preference analysis approach that we have applied in NegoManage system operates, similarly to the conjoint analysis approach, with complete offers and the corresponding score definitions (a full package must be specified and evaluated by the negotiators) we may face the problem of negotiator consistency in specifying different examples of offers and their scores. It may appear that two very similar packages are assigned to two separate indifference sets that differ much in terms of a linguistic utility evaluation or that the packages assigned to one indifference set differ too much to have assigned the same linguistic utility label. If so, we say that the problem of preference consistency occurs and consequently corrective actions need to be undertaken before the final scoring system is determined and used for the evaluation of offers in the actual negotiation phase.

In this paper we propose a simple mechanism for verifying the consistency of negotiators' preferences that we apply in the NegoManage system. The preference consistency check is based on the concept of the Jaccard index allowing for measuring the similarity between fuzzy sets. Since the NegoManage preference elicitation approach allows to obtain the characteristics of equivalence sets in the form of probability distributions, we may further consider these functions as fuzzy sets membership functions and use the distribution characteristics to compute the Jaccard index for every pair of equivalence sets elicited from the negotiator. The Jaccard indexes measure similarity between the indifference sets defined by the negotiator. If for any two sets the Jaccard index is too high, it is recommended to reconsider these two indifference sets by analyzing both the examples of offers constituting these sets and the values of utility scores assigned to these sets. It may appear that it would be reasonable to join the sets or differentiate their original scores to obtain a more accurate final scoring system of negotiation offers.

The paper consist of four more sections. In Section 1 we introduce the general idea of eliciting preferences of negotiators in the NegoManage system. Then in Section 2 we give a deeper insight into the method of defining the preferences by means of indifference sets that we proposed earlier in the NegoManage NSS and discuss the issue of kernel density analysis required for determining the main characteristics of these sets. We present also briefly the major idea of the Jaccard index (Section 3) and the possibility of inter-change between the two alternative approaches to describing the indifference sets, i.e. probability-based and possibility-based. In Section 4 we show in detail an example of analyzing the preference of the negotiator and measuring its consistency using a very simple negotiation problem where the template consists of three negotiation issues only. We conclude the paper with some comments on the proposed mechanism of preference elicitation and consistency verification.

1. Preference representation in NegoManage system – Indifference sets and their characteristics

In the NegoManage system the negotiator defines preferences by specifying several sets of alternatives, called indifference sets (surfaces), and assigning a degree of utility to each surface. Each indifference set consists of the alternatives that the negotiator considers to be equally good. The degree of utility assigned to the surface is chosen from a linguistic (verbal) scale [see Yevseyeva et al. 2008]. The scale is build on two levels. The first-level scale consists of seven verbal terms. First, the negotiator assigns to the indifference set a level from this scale. The second-level scale allows for stating precisely the degree of utility; namely by choosing a degree between two neighboring terms from the first scale. The second scale consists also of seven verbal terms and leads to an increase in the precision of the utility specification. Each linguistic utility level has its numeric equivalent used during the scoring procedure. However, such sets consisting of alternatives representing a particular level of utility may not be sufficient for deriving a full scoring system of the negotiation offers. There are probably other alternatives that may also belong to this surface, that were not specified by the negotiators in the preference elicitation stage but could easily be built in the actual negotiation phase by changing proportionally the resolution levels of the subsequent negotiation issues (making implicitly trade-offs between the negotiation issues). Initially the negotiators may not have specified all the salient alternatives for a particular indifference set because of lack of time, haste or simply because they subjectively felt that a certain alternative is not important (conveys no important information) in the definition of this indifference surface. Therefore we need to remember that the alternatives that comprise the indifference surfaces are only examples of a particular utility. There are many other alternatives (especially in the continuous negotiation problems) that may also belong to each of these sets. However, the degree of belonging to a surface may be partial since for alternatives not classified directly by the negotiator we may never be sure of their belonging to this surface. To cope with this type of uncertainty we propose to model the level (or chance) of belonging to the surface by using the notion of probability. More precisely, we propose to build a characteristic of an indifference surface in the form of probability distribution. Such a function will assign to each alternative a level of belonging to the particular indifference surface.

After the surfaces have been specified and the utility degrees have been assigned, the NegoManage system performs its most important task, namely the computation of probability distributions for all specified surfaces that will be further used to build a global scoring system. The probability assigned to a particular point of the indifference surface can be interpreted as a chance of actual assignment of this point to the indifference set. To build the characteristic of a surface we use the following straightforward postulate:

The closer an alternative under consideration is located to the one that fully belongs to the indifference surface, the higher is the level of probability that we assign to this alternative.

In other words, for an alternative located in the neighborhood of a fully classified alternative we first compute the distance to the classified alternative and map it into a similarity degree. The similarity degree of the considered alternative to the fully classified alternative is regarded as the probability of belonging to the surface. Based on our postulate we propose to build peaks around the classified alternatives as a preliminary step of building the probability distribution. Such peaks may have a bell shape of the normal distribution curves, and such a peak shape is considered at the current stage of research. However, there are no substantial or experimentally proved reasons for using this type of probability distributions in our approach. We simply use the normal distribution functions as the most commonly used solution in the selection of the predefined shape of the probability function since we lack the relevant information that would allow us to use other types of probability distribution functions. In the next step such peaks are fused together to form an overall multi-modal distribution which is treated as the indifference surface's characteristic together with utility levels assigned to the surfaces by the negotiator. This information about each indifference set is a basis for determining the score of any feasible alternative under evaluation later on in the actual negotiation phase, when the negotiators face the problem of scoring the offers presented by their counterpart as the negotiation compromise proposals.

2. Preference analysis in NegoManage system

Let us now look in detail at the formalism of the preference analysis approach implemented in the NegoManage system and described briefly in the previous section. Let us assume that the negotiator specifies a set of indifferent alternatives in the following form:

$$RS_i = \{\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n\} \tag{1}$$

where the indifference relationship holds between every pair of alternatives: $\overline{a}_i \approx \overline{a}_j$.

The utility value u_i assigned to the i-th indifference surface means that all alternatives in this set have this utility value since the alternatives in the set are equivalent in terms of preference, i.e. $\forall \overline{a} \in RS_i \mid u(\overline{a}) = u_i$. To illustrate this, let us consider a simple single-issue case. The focal negotiator decided to form the indifference surface by means of four alternatives.

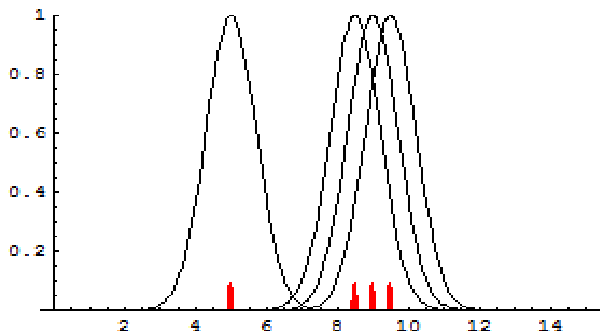

Figure 1. Alternatives and the corresponding peaks for defining the indifference surface (set)

In Figure 1 the peaks for four points are shown (red points indicate four alternatives belonging to the surface) that the negotiator decided to assign to the indifference surface under consideration. The peaks describe the probability distribution that this alternative and the similar ones (the ones in the close neighborhood) belong to the indifference surface under consideration. The concept of Kernel Density Estimation allows for deriving the overall distribution by fusing the peaks using an average operator. Assuming that the kernel is in the shape of normal distribution, the distribution for the surface specified above is of the following form:

$$f_{RS}(x) = \frac{1}{4d}\sum_{i=1}^{4}\exp\left(-\left(\frac{x-m_i}{d}\right)^2\right). \tag{2}$$

where m_i are the locations of the four points.

As the result of fusing the peaks into a compound distribution we obtain a distribution with two peaks (see Figure 2).

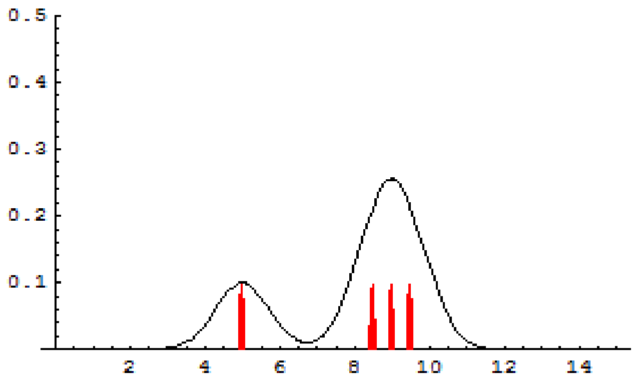

Figure 2. Aggregated peaks for defining the indifference surface (set)

The first peak is located around the first classified alternative and the second peak is located around the group of the other three. What we can observe here is the accumulation of high probability value around the group of three alternatives. We can conclude from this observation that points densely grouped in a small area can accumulate a higher probability in this area than the probability accumulated by other points. Consequently, for other regions represented by single alternatives the probability cumulated in the peaks around a single alternative may be decreased to a level which may be too low as compared to peaks located around dense groups of alternatives. Therefore, we propose to split the set of classified alternatives using hierarchical clustering into groups where the alternatives are close to each other, in order to build peaks over groups of alternatives instead of building peaks over single alternatives. Such a procedure will avoid the accumulation of high probabilities around dense groups of alternatives (see Figure 3).

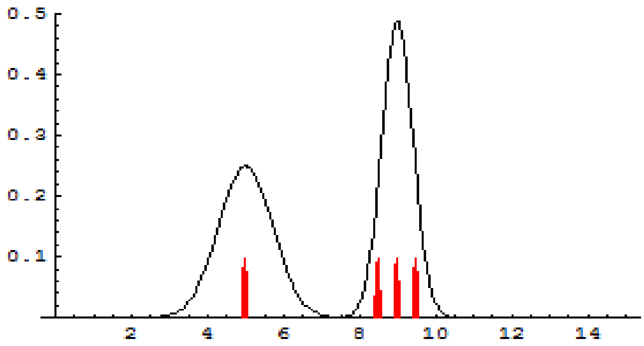

Figure 3. Aggregated peaks for the alternatives grouped within the indifference surface (set)

In a general multi-issue negotiation problem the probability distributions corresponding to the indifference surfaces are of multivariate form. Therefore, first we define the negotiation alternatives as follows. Every alternative $\overline{a}$ is described by a sequence of mappings $g_1, g_2, \ldots, g_m$ in the following way:

$$\overline{a} = (g_1(\overline{a}), g_2(\overline{a}), \ldots, g_m(\overline{a})) . \tag{3}$$

where each mapping g_s maps the alternative $\overline{a}$ into the numerical value of *s*th issue.

The simplest way to cluster the alternatives constituting the indifference set is to use hierarchical clustering [see Hartigan 1975; Hair and Black 1992]. The algorithm is agglomerative which means that at the beginning of the procedure each cluster consists of one alternative. In the next stages of the clustering algorithm the clusters are successively merged together. The number of clusters is decreasing while the size of clusters grows. The merging stops when the maximal distance between the alternatives inside the clusters reaches a selected level. As a result of this algorithm we obtain a split of the indifference surface. Given a split of the set RS_i into k disjoint subsets $M_{i1}, M_{i2}, \ldots, M_{ik}$, the means for all subsets (clusters) are computed: $\overline{m}_{i1}, \overline{m}_{i2}, \ldots, \overline{m}_{ik}$ (for the computation of the mean we use simple average). The multi-modal distribution is built over the indifference surface consisting of kernels determined over subsets M_{ij} [see Parzen 1962]. For the *j*th cluster of the *i*th indifference surface the multi-normal kernel distribution may be calculated:

$$f_{M_{ij}}(\overline{a}) = \frac{1}{(2\pi)^{k/2} |\Sigma_{ij}|^{1/2}} \exp\left(\frac{1}{2}(\overline{a} - \overline{m}_{ij})' \Sigma_{ij}{}^{-1} (\overline{a} - \overline{m}_{ij}) \right). \quad (4)$$

where Σ_{ij} is the covariance matrix.

Let us assume that $M_{ij} = \{\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n\}$. For the estimation of the covariance matrix we use the following estimator:

$$\Sigma_{ij} = \frac{1}{n-1} \sum_{l=1}^{n} (\overline{a}_l - \overline{m}_{ij})(\overline{a}_l - \overline{m}_{ij})'. \quad (5)$$

where the operator ' is the matrix transposition.

Having the distributions $f_{M_{ij}}$ for all clusters M_{ij} the final characteristics in the form of a multi-modal distribution is built and has the following form:

$$f_{RS_l}(\overline{a}) = \frac{1}{k} \sum_{j=1}^{k} f_{M_{ij}}(\overline{a}). \quad (6)$$

The final scoring system consists of the sequence of indifference sets distributions together with its utility (defined in a linguistic form) assigned to the indifference set by the negotiator:

$$(f_{RS_i}, u_i) \quad i \in \{1, \ldots, m\}. \quad (7)$$

During the actual negotiation phase the scoring system is used to evaluate the chosen alternative $\overline{a}$ in the following way. First the degree of belonging to a particular indifference set is computed. In other words, the probability of belonging to a particular indifference set is computed:

$$p(i) = p_i(\overline{a}) = f_{RS_i}(\overline{a}) \quad i \in \{1, \ldots, m\}. \quad (8)$$

The degree of belonging $p_i(\overline{a})$ is computed for all indifference sets. As a result we obtain a discrete probability distribution over a set of indices $i \in \{1, \ldots, m\}$ indexing all indifference surfaces (indifference sets). The distribution $p(i)$ tells us the degree of belonging of the given alternative to any indifference set. In the next step of computation we need to obtain the final utility for the alternative $\overline{a}$. In other words, we look for the indifference set to which the alternative $\overline{a}$ belongs with the highest degree. However, the notion of belonging to a set is not binary here. The alternative may belong to many indifference sets with different values of belonging degree. Therefore, to obtain the indifference set index and the final utility of the alternative $\overline{a}$ we use the concept of von Neumann-Morgenstern expected utility [see Neumann and Morgenstern 1944]. The linguistic utility in NegoManage consists of two linguistic values (v_{1i}, v_{2i}) describing the utility in terms of two integrated scales and these values correspond to numerical interval $[l_i, r_i]$:

$$u_i = (v_{1i}, v_{2i}) \rightarrow [l_i, r_i] \quad i \in \{1, \ldots, m\}. \tag{9}$$

These two boundary values of expected utility (lower and upper) are computed as follows:

$$LEU(\overline{a}) = \sum_{i=1}^{n} p_i(\overline{a}) \cdot l_i, \tag{10}$$

$$UEU(\overline{a}) = \sum_{i=1}^{n} p_i(\overline{a}) \cdot r_i. \tag{11}$$

As a result we obtain a pair of utilities describing the final utility interval $[LEU(\overline{a}), UEU(\overline{a})]$ that can be mapped back into linguistic utility to present it to the user. The precise description of interval in numerical form can be used in further computation.

3. Preference consistency and Jaccard index

The consistency of the scoring system is a key issue in the NegoManage system. Since one set contains alternatives ranked with a different degree of utility and a different indifference set contains alternatives ranked with different degree of utility, two difference sets with different utility values should not overlap (should be disjoint). If an alternative belonged to different indifference surfaces, it would mean that different levels of utility have been assigned to the same alternative. Therefore, we assume that the preference structure is fully consistent if all indifference surfaces are disjoint. However, if there is a partial overlap of two indifference surfaces, namely some alternatives partially belong to both surfaces, then a measure needs to be defined indicating the extent to which the condition of separation of two surfaces is violated. This extent is indicated in the simplest possible way by the number of alternatives belonging to both surfaces. However, we need to normalize this value to make the measure of inconsistency universal. The normalization is obtained by dividing the cardinality of the intersection of two surfaces by the cardinality of the union of these surfaces. If the intersection of two surfaces is non-empty, we will measure the preference inconsistency using the Jaccard index described above, given by the following formula:

$$J(A,B)=\frac{m(A\cap B)}{m(A\cup B)}. \tag{12}$$

where m is the cardinality of a set.

In the NegoManage system we have at our disposal the characteristics of surfaces given by probability distributions. We use the concept of probability to describe the degree of belonging of an alternative to the indifference surface. However, this interpretation can be also used if we want to describe the indifference surface in the form of a fuzzy set, namely with the membership degree stating the extent to which an alternative is included in the fuzzy indifference surface. Unfortunately, from a formal point of view, the probability distributions cannot be directly treated as membership functions of a fuzzy surface. One of the reasons for this is the normalization axiom defined in different way for a probability distribution and a possibility distribution (a fuzzy set concept used to describe the plausibility of belonging to the indifference surface in our application context). Namely, the normalization condition for the probability distributions means that the probabilities of all alternatives sum up to 1, and in the case of a possibility distribution the function reaches 1 for some alternative (which is the maximal distribution value). The following formula is an extension of the concept of the Jaccard index for fuzzy sets or possibility distributions:

$$J(A,B)=\frac{m(A\cap B)}{m(A\cup B)}=\frac{\max_{\overline{u}\in\Omega}\min(\mu_A(\overline{u}),\mu_B(\overline{u}))}{\max_{\overline{u}\in\Omega}\max(\mu_A(\overline{u}),\mu_B(\overline{u}))}. \tag{13}$$

where Ω is the space of all feasible alternatives, and μ_A, μ_B are the membership functions of the sets A and B.

To apply the fuzzy Jaccard coefficient in our particular application context, first we need to convert the surface characteristics given in the forms of probability distributions into possibility distributions. Such conversions have been proposed by Dubois et al. [2004]. The most important axiom distinguishing the possibility measures from probability measures is:

$$\forall A\subseteq X \quad \Pi(A)=\sup\{\pi(x), x\in A\}. \tag{14}$$

As we can see, this axiom involves the supremum operation instead of Riemann integration as it is done in the case of probability measures

$$\forall A \subseteq X \quad P(A) = \int_A p(x)dx \,. \tag{15}$$

Probability and possibility measures capture different facets of uncertainty. In the case of two disjoint subsets measured using probability measure, the measure of their union is equal to the sum of their measures. In the case of possibility measures the measure of union of disjoint subsets is the supremum (maximum in the case of finite sets). But some linkages between these two approaches may be distinguished (Dubois et al. 2004):

> *As it turns out, a numerical possibility measure, restricted to measurable subsets, can also be viewed as an upper probability function* [Dubois and Prade 1992]. *Formally, such a real-valued possibility measure P is equivalent to the family P(P) of probability measures such that* $P(\Pi) = \{P, \forall A \text{ measurable}, P(A) \leq \Pi(A)\}$

While converting a probability distribution to a possibility distribution the most important principle to be kept in mind is that introduced by Zadeh [1965], stating that *an event must be possible prior to being probable*. This principle is consistent with the fact that possibility distributions encode upper probability distributions. According to Dubois and Prade [1992] the relationship between the possibility distribution and its probability counterpart is described formally as the order preservation rule

$$\pi(x) < \pi(x') \quad \text{if and only if} \quad p(x) < p(x') \,. \tag{16}$$

Let us assume that we have a probability distribution defined over a finite set of alternatives: $\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n$. Moreover, without loss of generality the alternatives are ordered according to the probability values, namely we have the corresponding levels of probability: $p_1 \geq p_2 \geq \ldots \geq p_n$. We want to derive the corresponding possibility distribution satisfying the following assumptions

- $P(A) \leq \Pi(A) \quad \forall A \subseteq X$
- p and π are order-equivalent
- π is maximally specific (any other solution π' is such that $\pi \leq \pi'$).

Under these assumptions there exists a unique possibility distribution that can be obtained as follows

$$\pi_1 = 1,$$

$$\pi_i = \begin{cases} \sum_{j=i,n} p_j & if \quad p_{i-1} > p_i \\ \pi_{i-1} & otherwise \end{cases}. \tag{17}$$

All probability distributions characterizing the indifference surfaces in the NegoManage scoring system are converted to possibility distributions according to the procedure above. After the conversions the indifference surfaces can be compared using the fuzzy Jaccard index (formula 13) by taking for the comparison the obtained possibility distributions.

In the case of the fuzzy Jaccard coefficient the condition of preference consistency is of different nature, since all indifference surfaces described by possibility distributions overlap to some extent. Therefore, we define the so-called soft consistency conditions. The postulate for defining the consistency condition is: The higher the distance between the surfaces on the utility scale, the lower should be the overlap between these surfaces as computed using the fuzzy Jaccard index. Formally, the condition is defined as follows:

Given three indifference surfaces indexed with three values: *i*, *j*, *k*, and the utility scores corresponding to these surfaces: u_i, u_j, u_k, the following implication holds:

$$u_i > u_k \wedge u_i > u_j \wedge (u_i - u_k \geq u_i - u_j) \Rightarrow J(RS_i, RS_k) - J(RS_i, RS_j) \leq t. \tag{18}$$

where *t* is the indifference threshold equal to a small percentage of the utility space (for instance 0.15). This formula means that if the *k*th surface is more distant from the *i*th surface than the *j*th surface is from the *i*th surface, then the overlap of the *i*th and *k*th surfaces should be lower than the overlap of the *i*th and *j*th surfaces. If for all pairs of indifference surfaces the overlap levels in the form of Jaccard coefficients have been computed, we obtain a matrix consisting of the following elements:

$$M(i,j) = J(RS_i, RS_j)$$

Based on the values encoded by this matrix we can check if the preference consistency condition holds according to the formula (18).

4. Example of preference consistency analysis

Let us consider a simple problem of defining the negotiator's preferences and verifying their consistency in the NegoManage system. We assume that during the problem structuring process the negotiators decided to consider three negotiation issues, namely: price, delivery time and warranty. We will illustrate the preference analysis from the buyer's point of view. During the first stage of preference analysis the negotiator specified the following feasible ranges for the three negotiation issues:

– price: [20$, 80$],
– warranty: [2 months, 24 months],
– delivery time: [7 days, 21 days].

The preference analysis system maps the ranges of the issues into [0,1] intervals using the standard normalization formula. For the price, the mapping is:

$$g_1(\overline{a}) = \frac{80 - a_p}{80 - 20} = \frac{80 - a_p}{60}.$$

The mappings corresponding to warranty and delivery time are:

$$g_2(\overline{a}) = \frac{a_g - 2}{24 - 2} = \frac{a_g - 2}{22},$$

$$g_3(\overline{a}) = \frac{21 - a_d}{21 - 7} = \frac{21 - a_d}{14}.$$

After the system performed the mapping of all attributes, the user can use both scales. In the next stage of preference analysis the negotiator specifies the indifference surfaces. As an example, we consider here the first three indifference surfaces. Let us assume that the negotiator specified the first three surfaces (out of thirteen) as follows:

$\mathbf{RS_1}$ = {(0,0,0)}

$\mathbf{RS_2}$ = {(0.25, 0.0, 0.0),(0.0, 0.25, 0.0) ,(0.0, 0.0, 0.25)}

$\mathbf{RS_3}$ = {(0.0, 0.0, 0.5), (0.0, 0.25, 0.25), (0.25, 0.0, 0.25), (0.0, 0.5, 0.0), (0.25, 0.25, 0.0), (0.5, 0.0, 0.0)}

We will show now how the characteristics of the indifference surface are created by the NegoManage system using the example of the third indifference surface RS_3. Let us denote each alternative assigned to this surface by a_i, where i is the consecutive number of the alternative in the surface. Thus we have: a_1= (0.0, 0.0, 0.5), a_2 = (0.0, 0.25, 0.25), a_3 = (0.25, 0.0, 0.25), a_4 = (0.0, 0.5, 0.0), a_5=(0.25, 0.25, 0.0) and a_6=(0.5, 0.0, 0.0). We use hierarchical clustering to split the set RS_3 into clusters. According to the hierarchical clustering algorithm we begin with the initial partition consisting of single-element aggregations:

$$P_1=\{\{a_1\},\{a_2\},\{a_3\},\{a_4\},\{a_5\},\{a_6\}\}.$$

The distance matrix D_1 is computed in the following way. First we compute the means m_i for all defined clusters (in this case the clusters are the single elements a_i). Each element of the distance matrix D_1 is computed in the following way

$$D_1(i,j) = d_e(m_i, m_j), \tag{19}$$

where d_e is the Euclidean distance.
We obtain

$$D_1 = \begin{bmatrix} 0 & 0.35 & 0.43 & 0.7 & 0.75 & 0.86 \\ 0.35 & 0 & 0.25 & 0.35 & 0.43 & 0.61 \\ 0.43 & 0.25 & 0 & 0.43 & 0.35 & 0.43 \\ 0.7 & 0.35 & 0.43 & 0 & 0.25 & 0.5 \\ 0.75 & 0.43 & 0.35 & 0.25 & 0 & 0.25 \\ 0.86 & 0.61 & 0.43 & 0.5 & 0.25 & 0 \end{bmatrix}.$$

The value $D_1(2,3)$=0.25 is the smallest in the matrix D_1 (except for the diagonal elements which are not taken into account). Therefore, the elements a_2 and a_3 are merged in the next step of the clustering algorithm:

$$P_2=\{\{a_1\},\{a_2, a_3\},\{a_4\},\{a_5\},\{a_6\}\}.$$

We proceed this way using the notion of closest neighbor in calculating the distances between the clusters and finally obtain the following sequence of ascending partitions:

$$P_1=\{\{a_1\},\{a_2\},\{a_3\},\{a_4\},\{a_5\},\{a_6\}\},$$

$$P_2=\{\{a_1\},\{a_2, a_3\},\{a_4\},\{a_5\},\{a_6\}\},$$

$$P_3=\{\{a_1\},\{a_2, a_3\},\{a_4, a_5\},\{a_6\}\},$$

$$P_4=\{\{a_1\},\{a_2, a_3, a_4, a_5\},\{a_6\}\},$$

$$P_5=\{\{a_1\},\{a_2, a_3, a_4, a_5, a_6\}\},$$

$$P_6=\{\{a_1, a_2, a_3, a_4, a_5, a_6\}\}.$$

On the fusion level 0.5 we obtain the partition P_5 . Over this partition we will span the multi-modal distribution. For the partition P_5 we have two clusters:

$$M_1=\{a_1\},$$

$$M_2=\{a_2, a_3, a_4, a_5, a_6\},$$

with the following means:

$$m_1 = a_1 = (0, 0, 0.5),$$

$$m_2 = 0.2\ (a_2 + a_3 + a_4 + a_5 + a_6) = (0.2, 0.2, 0.1).$$

and the following sigma matrices:

$$\Sigma_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Sigma_2 = \begin{bmatrix} 0.035 & -0.0075 & -0.0075 \\ -0.0075 & 0.015 & 0.015 \\ -0.0075 & 0.015 & 0.015 \end{bmatrix}.$$

To avoid matrix singularity (the first matrix is singular since the first cluster contains only one element) we add a small value to the diagonal elements of the sigma matrices:

$$\overline{\Sigma}_1 = \Sigma_1 + 0.25E$$

$$\overline{\Sigma}_2 = \Sigma_2 + 0.25E$$

where E is the identity matrix.

The resulting probability density functions for two kernels forming the final probability density function are of the following form:

$$f_{M_1}(\overline{a}) = 12.7389 \exp\left(\frac{1}{2}(\overline{a} - [0,0.5,0.5])' \begin{bmatrix} 0.025 & 0 & 0 \\ 0 & 0.025 & 0 \\ 0 & 0 & 0.025 \end{bmatrix} (a - [0,0.5,0.5]) \right)$$

$$f_{M_2}(\overline{a}) = 6.46301 \exp\left(\frac{1}{2}(\overline{a} - [0.2,0.1,0.1])' \begin{bmatrix} 0.06 & -0.0075 & -0.075 \\ -0.075 & 0.04 & 0.015 \\ -0.0075 & 0.015 & 0.265 \end{bmatrix} (a - [0.2,0.1,0.1]) \right)$$

The final function for the indifference set RS_3 is represented by:

$$f_{RS_3}(\overline{a}) = \frac{1}{2}(f_{M_1}(\overline{a}) + f_{M_2}(\overline{a})).$$

Similarly, the probability density functions are calculated for all thirteen remaining equivalence sets.

Having all the indifference sets described by distribution functions we can verify the consistency of their definition provided by the negotiator. As said in the previous section, to check the consistency we will use the Jaccard index, that measures the similarity between two equivalence sets. The higher the similarity between two equivalence sets, the less consistent are the preferences. In our example the matrix of Jaccard indices is:

$$\begin{pmatrix}
0.99 & 0.99 & 0.84 & 0.77 & 0.89 & 0.83 & 0.77 & 0.63 & 0.2 & 0.1 & 0.04 & 0.01 & 0.02 \\
0.99 & 0.99 & 0.84 & 0.7 & 0.95 & 0.67 & 0.49 & 0.29 & 0.07 & 0.02 & 0. & 0. & 0.01 \\
0.84 & 0.84 & 1. & 0.99 & 0.86 & 0.91 & 0.85 & 0.85 & 0.16 & 0.07 & 0.02 & 0. & 0.03 \\
0.77 & 0.7 & 0.99 & 0.99 & 0.88 & 0.91 & 0.97 & 0.9 & 0.28 & 0.2 & 0.07 & 0.02 & 0.07 \\
0.89 & 0.95 & 0.86 & 0.88 & 0.99 & 0.88 & 0.95 & 0.88 & 0.79 & 0.74 & 0.19 & 0.06 & 0.14 \\
0.83 & 0.67 & 0.91 & 0.91 & 0.88 & 0.99 & 0.94 & 0.98 & 0.88 & 0.71 & 0.78 & 0.23 & 0.45 \\
0.77 & 0.49 & 0.85 & 0.97 & 0.95 & 0.94 & 1. & 0.99 & 0.93 & 0.85 & 0.84 & 0.51 & 0.63 \\
0.63 & 0.29 & 0.85 & 0.9 & 0.88 & 0.98 & 0.99 & 1. & 0.88 & 0.85 & 0.89 & 0.64 & 0.77 \\
0.2 & 0.07 & 0.16 & 0.28 & 0.79 & 0.88 & 0.93 & 0.88 & 0.99 & 0.89 & 0.82 & 0.93 & 0.93 \\
0.1 & 0.02 & 0.07 & 0.2 & 0.74 & 0.71 & 0.85 & 0.85 & 0.89 & 0.99 & 0.85 & 0.73 & 0.75 \\
0.04 & 0. & 0.02 & 0.07 & 0.19 & 0.78 & 0.84 & 0.89 & 0.82 & 0.85 & 1. & 0.77 & 0.75 \\
0.01 & 0. & 0. & 0.02 & 0.06 & 0.23 & 0.51 & 0.64 & 0.93 & 0.73 & 0.77 & 1. & 0.96 \\
0.02 & 0.01 & 0.03 & 0.07 & 0.14 & 0.45 & 0.63 & 0.77 & 0.93 & 0.75 & 0.75 & 0.96 & 0.99
\end{pmatrix}$$

All Jaccard indices this matrix are obtained as follows:

$$M(i,j) = J(RS_i, RS_j) = \frac{\max_{\overline{a}\in D}(\min(\pi_i(\overline{a}), \pi_j(\overline{a}))}{\max_{\overline{a}\in D}(\max(\pi_i(\overline{a}), \pi_j(\overline{a}))}$$

where the functions π_i , π_j are the possibility distributions corresponding to two indifference surfaces obtained by the transformation of probability distributions f_{RS_i} , f_{RS_j} also corresponding to the two given surfaces. As we can see from the above matrix the closer two surfaces are located to each other in terms of the utility levels, the higher are the values of the corresponding Jaccard indices. For instance, for the second and first surfaces the Jaccard value is 0.99 which is very high since these surfaces are close to each other. If we take a look at a selected matrix row, we can see that if we move right from the diagonal element the values are weakly decreasing (with an accuracy to the indifference threshold equal to 0.15). Analogously, if we move left along the row from the diagonal element the values are also weakly decreasing (with an accuracy to the indifference threshold equal to 0.15). The same observation holds if we move along a column up or down from the diagonal element. In this example we have defined surfaces preserving the preference consistency condition in terms of crisp definitions of the Jaccard index. If two indifference surfaces in a crisp form are disjoint (consistency condition holds – Jaccard index is equal to 0) its fuzzy counterparts (fuzzy surfaces) should in result have low level of overlap when the fuzzy Jaccard index is used for the comparison of surfaces (fuzzy Jaccard index should be low).

Conclusions

In this paper we presented a straightforward method for checking the negotiator's preference consistency for the preference elicitation method based on the notion of indifference sets, applied in the NegoManage system, that we have built and developed beforehand [see Brzostowski and Wachowicz 2009, 2010]. It seems vital to verify whether the negotiator defines preferences in a coherent and consistent way in every decision problem, but especially when the preference elicitation process has a decompositional character, i.e. the preferences are derived from the examples of the predefined complete packages and evaluated by the negotiator in the prenegotiation phase. In this approach it may very often appear that while defining the examples, the negotiator builds two very similar examples of negotiation offers but assigns them to two indifferent sets with different utility scores. If such a situation occurs, the scoring system of negotiation offers derived from the predefined examples appears imprecise and may result in the false scorings determined for the negotiation offers under evaluation in the actual negotiation phase. If so, the negotiator may feel that the whole scoring system is not adequate to her/his subjective and intrinsic preferences that she/he tried to define in prenegotiation. To avoid such a situation we recommend to check the consistency of preferences just after the definition of the examples of offers and the determination of the characteristics of indifference sets given in the form of distribution functions. We decided to use the simplest possible solution, which is to apply the Jaccard index. Given two sets, the Jaccard index compares the number of alternatives that may be assigned to both sets with the number of alternatives assigned to each set separately. However, while describing the indifference sets we operate with probability distribution functions, therefore we tried to give a rationale to move to the concept of possibility and used the Jaccard index formula defined for the fuzzy sets or possibility distributions. This simple mechanism allows us to find the sets that are too similar and ask the negotiator to revise their definitions. If two similar indifference sets are identified, the negotiator may change their forms by moving or eliminating some sample offers within the sets. She/he may also decide to join these two sets if necessary and assign to them a new value of the linguistic utility. After the negotiator's revision the consistency checkup is conducted once again to verify the impact of these changes on the form and quality of the new scoring system of negotiation offers.

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References

Behzadian M., Kazemzadeh R.B., Albadvi A., Aghdasi M. (2010), *PROMETHEE: A Comprehensive Literature Review on Methodologies and Applications*, "European Journal of Operational Research", No. 200(1).

Brans J. (1982), *L'ingéniérie de la décision. Elaboration d'instruments d'aide à la décision. Méthode PROMETHEE*, Universite Laval, Québec.

Brzostowski J., Wachowicz T. (2009), *Conceptual Model of eNS For Supporting Preference Elicitation and Counterpart Analysis*, [in:] *Proceedings of GDN 2009: An International Conference on Group Decision and Negotiation*, eds. D.M. Kilgour, Q. Wang, Wilfried Laurier University, Waterloo.

Brzostowski J., Wachowicz T. (2010), *Building Personality Profile Of Negotiator For Electronic Negotiations*, [in:] *Multiple Criteria Decision Making '09*, eds. T. Trzaskalik, T. Wachowicz, The Publisher of The University of Economics, Katowice.

Dubois D., Prade H. (1992), *When upper Probabilities are Possibility Measures*, "Fuzzy Sets and Systems", No. 49.

Dubois D., Foulloy L., Mauris G., Prade H. (2004), *Probability-possibility Transformations, Triangular Fuzzy Sets and Probabilistic Inequalities*, "Reliable Computing", No. 10.

Figuera J., Greco S., Ehrgott M. (eds.), 2005, *Multiple Criteria Decision Analysis. State of the Art Surveys*, Springer Science + Business Media, Boston.

Hair J.F., Black B. (1992), *Multivariate Data Analysis*, Macmillan, New York.

Hartigan J. (1975), *Clustering Algorithms*, Wiley.

Keeney R., Raiffa H. (1993), *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*, Cambridge University Press, New York.

Kersten G.E, Noronha S.J. (1999), *WWW-based Negotiation Support: Design, Implementation and Use*, "Decision Support Systems", No. 25(2).

Neumann J. von, Morgenstern O. (1944), *Theory of Games and Economic Behavior*, Princeton Univerosty Press, Princeton.

Omkarprasad S.V., Sushil K. (2006), Analytic Hierarchy Process: An Overview of Applications, "European Journal of Operational Research", No. 169(1).

Paradis N., Gettinger J., Lai H., Surboeck M., Wachowicz T. (2010), *E-Negotiations via Inspire 2.0: The System, Users, Management and Projects*, [in:] *Group Decision and Negotiations 2010. Proceedings. The Center for Collaboration Science*, ed. G.J. de Vreede, University of Nebraska at Omaha.

Parzen E. (1962), *On Estimation of a Probability Density Function and Mode*, "Ann. Math. Stat." 33.

Roy B., Bouyssou D., (1993), *Aide multicritere a la decision: Methodes et cas*, Economica, Paris.

Saaty T.L., Alexander J.M. (1989), *Conflict Resolution: The Analytic Hierarchy Approach*, Praeger, New York.

Saaty T. (1980), *The Analytic Hierarchy Process*, McGraw Hill, New York.

Schoop M., Jertila A., List T. (2003), *Negoisst: A Negotiation Support System for Electronic Business-to Business Negotiations in Ecommerce*, "Data and Knowledge Engeneering", No. 47.

Thiessen E.M., Soberg A. (2003), *Smartsettle Described with the Montreal Taxonomy*, "Group Decision and Negotiation", 12.

Wachowicz T., Kersten G.E. (2009), *Zachowania i decyzje negocjacyjne uczestników negocjacji elektronicznych*, [w:] *Człowiek i jego decyzje*, tom 2, red. K.A. Kłosiński, A. Biela, Wydawnictwo KUL, Lublin.

Zadeh Z.A. (1965), *Fuzzy Sets*, "Information and Control", No. 8.

Yevseyeva I., Miettinen K., Räsänen P. (2008), *Verbal Ordinal Classification with Multicriteria Decision Aiding*, "European Journal of Operational Research", No. 185.